OCB: A Block-Cipher Mode of Operation for Efficient Authenticated Encryption

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August 3, 2001

Abstract

We describe a parallelizable block-cipher mode of operation that simultaneously provides privacy and authenticity. OCB encrypts-and-authenticates a nonempty string $M \in \{0,1\}^*$ using $\lceil |M|/n \rceil + 2$ block-cipher invocations, where n is the block length of the underlying block cipher. Additional overhead is small. OCB refines a scheme, IAPM, suggested by Jutla [20]. Desirable properties of OCB include: the ability to encrypt a bit string of arbitrary length into a ciphertext of minimal length; cheap offset calculations; cheap session setup, a single underlying cryptographic key; no extended-precision addition; a nearly optimal number of block-cipher calls; and no requirement for a random IV. We prove OCB secure, quantifying the adversary's ability to violate privacy or authenticity in terms of the quality of the block cipher as a pseudorandom permutation (PRP) or as a strong PRP, respectively.

Keywords: AES, authenticity, block ciphers, cryptography, encryption, integrity, modes of operation, provable security, standards .

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1 Introduction

Background. An authenticated-encryption scheme is a shared-key encryption scheme whose goal is to provide *both* privacy *and* authenticity. The encryption algorithm takes a key, a plaintext, and a nonce, and it returns a ciphertext. The decryption algorithm takes a key, a ciphertext, and a nonce, and it returns either a plaintext or a special symbol, Invalid. In addition to the customary privacy goal, an authenticated-encryption scheme aims for authenticity: if an adversary should try to create some new ciphertext, the decryption algorithm will almost certainly regard it as Invalid.

An authenticated-encryption scheme can be constructed by appropriately combining an encryption scheme and a message authentication code (MAC), an approach used pervasively in practice and in standards. (Analyses of these methods are provided in [6, 23]). But an extremely attractive goal is an authenticated-encryption scheme having computational cost significantly lower than the cost to encrypt plus the cost to MAC. The classical approach for trying to do this is to encrypt-with-redundancy, where one appends a noncryptographic checksum to the message before encrypting it, typically with CBC mode. Many such schemes have been broken. Recently, however, Jutla has proposed two authenticated-encryption schemes supported by a claim of provable security [20]. Virgil Gligor and Pompiliu Donescu have described a different authenticated-encryption scheme [14]. We continue in this line of work.

OCB mode. This paper describes a new mode of operation, OCB, which refines one of Jutla's schemes, IAPM [20]. OCB (which stands for "offset codebook") retains the principal characteristics of IAPM: it is fully parallelizable and adds minor overhead compared to conventional, privacy-only modes. But OCB combines the following features:

- *Arbitrary-length messages + minimal-length ciphertexts.* Any string $M \in \{0,1\}^*$ can be encrypted; $|M|$ need not be a multiple of the block length n. What is more, plaintexts are not padded to strings of length a multiple of n, and thus ciphertexts are as short as possible.
- *Nearly optimal number of block-cipher calls*: OCB uses $\lceil |M|/n \rceil + 2$ block-cipher invocations. (This count does not include a block-cipher call assumed to be made during session setup.) It is possible to make do with $\lceil |M|/n \rceil + 1$, but such alternatives scheme would be more complex or would require a random IV. Keeping low the number of block-cipher calls is especially important when messages are short. In many domains, short messages dominate.
- *Minimal requirements on nonces*: Like other encryption modes, OCB requires a nonce. The nonce must be non-repeating (the entity that encrypts chooses a new nonce for every message with the only restriction that no nonce is used twice) but it does not have to be unpredictable. Requiring of a nonce only that it be non-repeating is less error prone, and often more efficient, than requiring it to be unpredictable.
- *Improved offset calculations*: As with [14, 20], we require a sequence of *offsets*. We generate these in a particularly cheap way, each offset requiring just a few machine cycles. We avoid the use of extended-precision addition, which would introduce endian dependency and might make the scheme less attractive for dedicated hardware.
- *Single underlying key*: The key used for OCB is a single block-cipher key, and all block-cipher invocations are keyed by this one key, saving space and key-setup time.

Achieving the properties above has required putting together a variety of "tricks" that work together in just the right way. Many earlier versions of the algorithm were rejected because attacks were found or a proof could not be pushed through. We have found schemes of this sort to be amazingly "fragile"—tweak them a little and they break. We have concluded that, if the goals above are ever to be sought, they must be carefully addressed from the start.

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Performance. On a Pentium III processor, experiments show that OCB is about 6.5% slower than the privacy-only mode CBC. The cost of OCB is about 54% of the cost of CBC encryption combined with the CBC MAC. These figures assume a block cipher of AES128 [33].

In settings where there is adequate opportunity for parallelism, OCB will be faster than CBC. Parallelizability is important for obtaining the highest speeds from special-purpose hardware, and it may become useful on commodity processors. For special-purpose hardware, one may want to encrypt-and-authenticate at speeds near 10 Gbits/second—an impossible task, with today's technology, for modes like CBC encryption and the CBC MAC. (One could always create a mode that interleaves message blocks fed into separate CBC encryption or CBC MAC calculations, but that would be a new mode, and one with many drawback.) For commodity processors, there is an architectural trend towards highly pipelined machines with multiple instruction pipes and lots of registers. Optimally exploiting such features necessitates algorithms with plenty to do in parallel.

Security properties. We prove OCB secure, in the sense of reduction-based cryptography. Specifically, we prove indistinguishability under chosen-plaintext attack [2, 15] and authenticity of ciphertexts [6, 7, 21]. As shown in [6, 21], this combination implies indistinguishability under the strongest form of chosen-ciphertext attack (CCA) (which, in turn, is equivalent to non-malleability [9] under CCA [3, 22]). Our proof of privacy assumes that the underlying block cipher is good in the sense of a pseudorandom permutation (PRP) [5, 25], while our proof of authenticity assumes that the block cipher is a strong PRP [25]. The actual results are quantitative; the security analysis is in the concrete-security paradigm. The proofs use standard techniques, but pushed quite far.

We emphasize that OCB has stronger security properties than standard modes. In particular, non-malleability and indistinguishability under CCA are not achieved by CBC, or by any other standard mode, but these properties are achieved by OCB. We believe that the lack of strong security properties has been a problem for the standard modes of operation, because many users of encryption implicitly assume these properties when designing their protocols. For example, it is common to see protocols which use symmetric encryption in order to "bind together" the parts of a plaintext, or which encrypt related messages as a way to do a "handshake." Standard modes do not support such practices. This fact has sometimes led practitioners to invent or select peculiar ways to encrypt (a well-known example being the use of PCBC mode [26] in Kerberos v.4 [28]). We believe that a mode like OCB is less likely to be misused in applications because the usual abuses of privacy-only encryption become correct cryptographic techniques.

By way of comparison, a chosen-ciphertext attack by Bleichenbacher on the public-key encryption scheme of RSA PKCS #1, v.1, motivated the company that controls this de facto standard to promptly upgrade its scheme [8, 27]. In contrast, people seem to accept as a matter of course symmetric encryption schemes which are not even non-malleable (a weaker property than chosen-ciphertext security). There would seem to be no technical reason to account for this difference in expectations.

The future. We believe that *most* of the time privacy is desired, authenticity is too. As a consequence, fast authenticated encryption may quickly catch on. OCB has already appeared in one draft standard—the wireless LAN standard IEEE 802.11—and it is also under consideration by NIST.

2 Preliminaries

Notation. If a and b are integers, $a \leq b$, then $[a..b]$ is the set $\{a, a+1, \ldots, b\}$. If $i \geq 1$ is an

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integer then $\mathsf{ntz}(i)$ is the number of trailing 0-bits in the binary representation of i (equivalently, $\mathsf{ntz}(i)$ is the largest integer z such that 2^z divides i). So, for example, $\mathsf{ntz}(7) = 0$ and $\mathsf{ntz}(8) = 3$.

A *string* is a finite sequence of symbols, each symbol being 0 or 1. The string of length 0 is called the *empty string* and is denoted ε. Let $\{0,1\}^*$ denote the set of all strings. If $A, B \in \{0,1\}^*$ then $A\,B$, or $A \parallel B$, is their concatenation. If $A \in \{0,1\}^*$ and $A \neq \varepsilon$ then $\mathsf{firstbit}(A)$ is the first bit of A and $\mathsf{lastbit}(A)$ is the last bit of A. Let i, n be nonnegative integers. Then 0^i and 1^i denote the strings of i 0's and 1's, respectively. Let $\{0,1\}^n$ denote the set of all strings of length n. If $A \in \{0,1\}^*$ then $|A|$ denotes the length of A, in bits, while $\|A\|_n = \max\{1, \lceil |A|/n \rceil\}$ denotes the length of A in n-bit blocks, where the empty string counts as one block. For $A \in \{0,1\}^*$ and $|A| \le n$, $\mathsf{zpad}_n(A)$ is the string $A\,0^{n-|A|}$. With n understood we will write $A\,0^*$ for $\mathsf{zpad}_n(A)$. If $A \in \{0,1\}^*$ and $\tau \in [0..|A|]$ then $A\,[\text{first } \tau \text{ bits}]$ and $A[\text{last } \tau \text{ bits}]$ denote the first τ bits of A and the last τ bits of A, respectively. Both of these values are the empty string if $\tau = 0$. If $A, B \in \{0,1\}^*$ then $A \oplus B$ is the bitwise xor of $A\,[\text{first } \ell \text{ bits}]$ and $B\,[\text{first } \ell \text{ bits}]$, where $\ell = \min\{|A|, |B|\}$ (where $\varepsilon \oplus \varepsilon = \varepsilon$). So, for example, $1001 \oplus 11 = 01$. If $A = a_{n-1} \cdots a_1 a_0 \in \{0,1\}^n$ then $\mathsf{str2num}(A)$ is the number $\sum_{i=0}^{n-1} 2^i a_i$. If $a \in [0..2^n - 1]$ then $\mathsf{num2str}_n(a)$ is the n-bit string A such that $\mathsf{str2num}(A) = a$. Let $\mathrm{len}_n(A) = \mathsf{num2str}_n(|A|)$. We omit the subscript when n is understood.

If $A = a_{n-1} a_{n-2} \cdots a_1 a_0 \in \{0,1\}^n$ then $A {\ll} 1$ is the n-bit string $a_{n-2} a_{n-3} \cdots a_1 a_0 0$ which is a left shift of A by one bit (the first bit of A disappearing and a zero coming into the last bit), while $A {\gg} 1$ is the n-bit string $0 a_{n-1} a_{n-2} \ldots a_2 a_1$ which is a right shift of A by one bit (the last bit disappearing and a zero coming into the first bit).

In pseudocode we write "Partition M into $M[1] \cdots M[m]$" as shorthand for "Let $m = \|M\|_n$ and let $M[1], \ldots, M[m]$ be strings such that $M[1] \cdots M[m] = M$ and $|M[i]| = n$ for $1 \le i < m$." We write "Partition $\mathcal{C}$ into $C[1] \cdots C[m]T$" as shorthand for "if $|\mathcal{C}| < \tau$ then return Invalid. Otherwise, let $C = \mathcal{C}\,[\text{first } |\mathcal{C}| - \tau \text{ bits}]$, let $T = \mathcal{C}[\text{last } \tau \text{ bits}]$, let $m = \|C\|_n$, and let $C[1], \ldots, C[m]$ be strings such that $C[1] \cdots C[m] = C$ and $|C[i]| = n$ for $1 \le i < m$. Recall that $\|M\|_n = \max\{1, \lceil |M|/n \rceil\}$, so the empty string partitions into $m = 1$ block, that one block being the empty string.

The field with 2^n points. Let $\mathrm{GF}(2^n)$ denote the field with 2^n points. We interchangeably think of a point a in $\mathrm{GF}(2^n)$ in any of the following ways: (1) as an abstract point in a field; (2) as an n-bit string $a_{n-1} \ldots a_1 a_0 \in \{0,1\}^n$; (3) as a formal polynomial $a(\mathtt{x}) = a_{n-1}\mathtt{x}^{n-1} + \cdots + a_1\mathtt{x} + a_0$ with binary coefficients; (4) as an integer between 0 and $2^n - 1$, where the string $a \in \{0,1\}^n$ corresponds to the number $\mathsf{str2num}(a)$. For example, one can regard the string $a = 0^{125}101$ as a 128-bit string, as the number 5, as the polynomial $\mathtt{x}^2 + 1$, or as an abstract point in $\mathrm{GF}(2^{128})$. We write $a(\mathtt{x})$ instead of a if we wish to emphasize that we are thinking of a as a polynomial.

To add two points in $\mathrm{GF}(2^n)$, take their bitwise xor. We denote this operation by $a \oplus b$. To multiply two points in the field, first fix an irreducible polynomial $p_n(\mathtt{x})$ having binary coefficients and degree n: say the lexicographically first polynomial among the irreducible degree n polynomials having a minimum number of coefficients. For $n = 128$, the indicated polynomial is $p_{128}(\mathtt{x}) = \mathtt{x}^{128} + \mathtt{x}^7 + \mathtt{x}^2 + \mathtt{x} + 1$. To multiply $a, b \in \mathrm{GF}(2^n)$, which we denote $a \cdot b$, regard a and b as polynomials $a(\mathtt{x}) = a_{n-1}\mathtt{x}^{n-1} + \cdots + a_1\mathtt{x} + a_0$ and $b(\mathtt{x}) = b_{n-1}\mathtt{x}^{n-1} + \cdots + b_1\mathtt{x} + b_0$, form their product $c(\mathtt{x})$ over $\mathrm{GF}(2)$, and take the remainder one gets when dividing $c(\mathtt{x})$ by $p_n(\mathtt{x})$.

It is computationally simple to multiply $a \in \{0,1\}^n$ by $\mathtt{x}$. We illustrate the method for $n = 128$, in which case multiplying $a = a_{n-1} \cdots a_1 a_0$ by $\mathtt{x}$ yields $a_{n-1}\mathtt{x}^n + a_{n-2}\mathtt{x}^{n-1} + a_1\mathtt{x}^2 + a_0\mathtt{x}$. Thus, if the first bit of a is 0, then $a \cdot \mathtt{x} = a {\ll} 1$. If the first bit of a is 1 then we must add $\mathtt{x}^{128}$ to $a {\ll} 1$. Since $p_{128}(\mathtt{x}) = \mathtt{x}^{128} + \mathtt{x}^7 + \mathtt{x}^2 + \mathtt{x} + 1 = 0$ we know that $\mathtt{x}^{128} = \mathtt{x}^7 + \mathtt{x}^2 + \mathtt{x} + 1$, so adding $\mathtt{x}^{128}$ means to xor by $0^{120}10000111$. In summary, when $n = 128$,

$$a \cdot \mathtt{x} \;=\; \begin{cases} a {\ll} 1 & \text{if } \mathsf{firstbit}(a) = 0 \\ (a {\ll} 1) \oplus 0^{120}10000111 & \text{if } \mathsf{firstbit}(a) = 1 \end{cases}$$

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It is similarly easy to divide $a \in \{0,1\}^{128}$ by $\mathtt{x}$ (i.e., to multiply a by the multiplicative inverse of $\mathtt{x}$). If the last bit of a is 0, then $a \cdot \mathtt{x}^{-1}$ is $a \gg 1$. If the last bit of a is 1 then we must add (xor) to $a \gg 1$ the value $\mathtt{x}^{-1}$. Since $\mathtt{x}^{128} = \mathtt{x}^7 + \mathtt{x}^2 + \mathtt{x} + 1$ we have that $\mathtt{x}^{-1} = \mathtt{x}^{127} + \mathtt{x}^6 + \mathtt{x} + 1 = 10^{120}1000011$. In summary, when $n = 128$,

$$a \cdot \mathtt{x}^{-1} \;=\; \begin{cases} a \gg 1 & \text{if } \mathsf{lastbit}(a) = 0 \\ (a \gg 1) \oplus 10^{120}1000011 & \text{if } \mathsf{lastbit}(a) = 1 \end{cases}$$

Note that *huge* $= \mathtt{x}^{-1}$ is a large number (when viewed as such); in particular, it starts with a 1 bit, so *huge* $\geq 2^{n-1}$.

If $L \in \{0,1\}^n$ and $i \geq -1$, we write $L(i)$ as shorthand for $L \cdot \mathtt{x}^i$. Using the equations already given, we have an easy way to compute from L the values $L(-1)$, $L(0)$, $L(1), \ldots, L(\mu)$, where μ is small number.

Gray codes. For $\ell \geq 1$, a Gray code is an ordering $\gamma^\ell = (\gamma_0^\ell \;\; \gamma_1^\ell \;\; \ldots \;\; \gamma_{2^\ell-1}^\ell)$ of $\{0,1\}^\ell$ such that successive points differ (in the Hamming sense) by just one bit. For n a fixed number, OCB makes use of the "canonical" Gray code $\gamma = \gamma^n$ constructed by $\gamma^1 = (0\ 1)$ and, for $\ell > 0$,

$$\gamma^{\ell+1} = (\; 0\gamma_0^\ell \quad 0\gamma_1^\ell \quad \cdots \quad 0\gamma_{2^\ell-2}^\ell \quad 0\gamma_{2^\ell-1}^\ell \quad 1\gamma_{2^\ell-1}^\ell \quad 1\gamma_{2^\ell-2}^\ell \quad \cdots \quad 1\gamma_1^\ell \quad 1\gamma_0^\ell \;)$$

It is easy to see that γ is a Gray code. What is more, for $1 \leq i \leq 2^n - 1$, $\gamma_i = \gamma_{i-1} \oplus (0^{n-1}1 \ll \mathsf{ntz}(i))$. This makes it easy to compute successive points.

We emphasize the following characteristics of the Gray-code values $\gamma_1, \gamma_2, \ldots, \gamma_{2^n-1}$: that they are distinct and different from 0; that $\gamma_1 = 1$; and that $\gamma_i < 2i$.

Let $L \in \{0,1\}^n$ and consider the problem of successively forming the strings $\gamma_1 \cdot L$, $\gamma_2 \cdot L$, $\gamma_3 \cdot L, \ldots, \gamma_m \cdot L$. Of course $\gamma_1 \cdot L = 1 \cdot L = L$. Now, for $i \geq 2$, assume one has already produced $\gamma_{i-1} \cdot L$. Since $\gamma_i = \gamma_{i-1} \oplus (0^{n-1}1 \ll \mathsf{ntz}(i))$ we know that

$$\begin{aligned} \gamma_i \cdot L &= (\gamma_{i-1} \oplus (0^{n-1}1 \ll \mathsf{ntz}(i))) \cdot L \\ &= (\gamma_{i-1} \cdot L) \oplus (0^{n-1}1 \ll \mathsf{ntz}(i)) \cdot L \\ &= (\gamma_{i-1} \cdot L) \oplus (L \cdot \mathtt{x}^{\mathsf{ntz}(i)}) \\ &= (\gamma_{i-1} \cdot L) \oplus L(\mathsf{ntz}(i)) \end{aligned}$$

That is, the ith word in the sequence $\gamma_1 \cdot L, \gamma_2 \cdot L, \gamma_3 \cdot L, \ldots$ is obtained by xoring the previous word with $L(\mathsf{ntz}(i))$. Had the sequence we were considering been $\gamma_1 \cdot L \oplus R$, $\gamma_2 \cdot L \oplus R$, $\gamma_3 \cdot L \oplus R, \ldots$, the ith word would be formed in the same way for $i \geq 2$, but the first word in the sequence would have been $L \oplus R$ instead of L.

3 The Scheme

Parameters. To use OCB one must specify a block cipher and a tag length. The *block cipher* is a function $E : \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$, for some number n, where each $E(K, \cdot) = E_K(\cdot)$ is a permutation on $\{0,1\}^n$. Here $\mathcal{K}$ is the set of possible keys and n is the block length. Both are arbitrary, though we insist that $n \geq 64$, and we discourage $n < 128$. The *tag length* is an integer $\tau \in [0..n]$. By trivial means, the adversary will be able to forge a valid ciphertext with probability $2^{-\tau}$. The popular block cipher to use with OCB is likely to be AES [33]. As for the tag length, a suggested default

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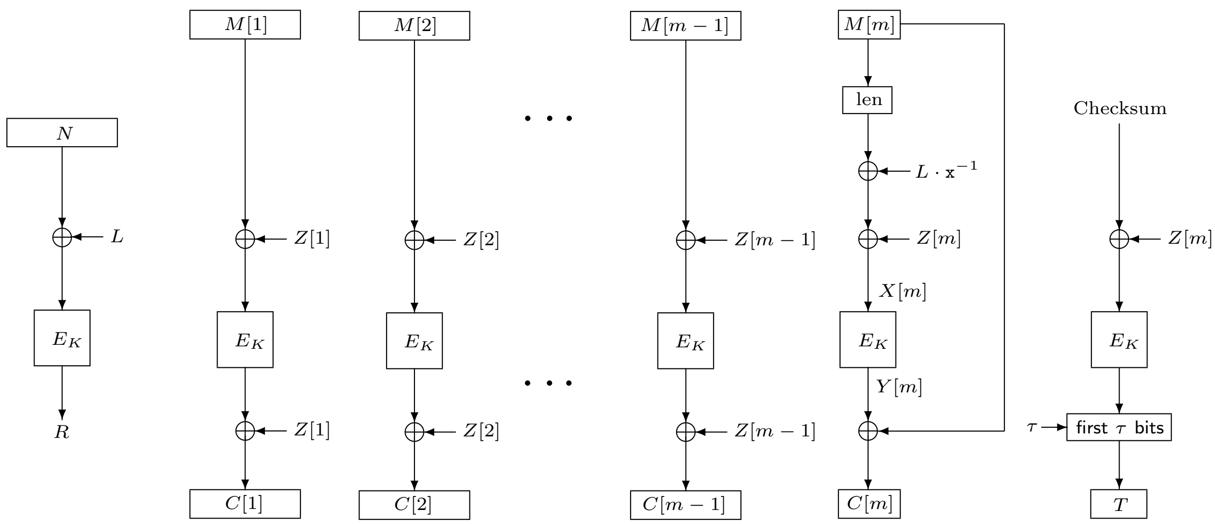

Algorithm OCB.Enc$_K$ (N, M)

Partition M into $M[1] \cdots M[m]$
$L \leftarrow E_K(0^n)$
$R \leftarrow E_K(N \oplus L)$
for $i \leftarrow 1$ **to** m **do** $Z[i] = \gamma_i \cdot L \oplus R$
for $i \leftarrow 1$ **to** $m-1$ **do**
$\quad C[i] \leftarrow E_K(M[i] \oplus Z[i]) \oplus Z[i]$
$X[m] \leftarrow \mathrm{len}(M[m]) \oplus L \cdot \mathtt{x}^{-1} \oplus Z[m]$
$Y[m] \leftarrow E_K(X[m])$
$C[m] \leftarrow Y[m] \oplus M[m]$
$C \leftarrow C[1] \cdots C[m]$
Checksum $\leftarrow$
$\quad M[1] \oplus \cdots \oplus M[m-1] \oplus C[m]\,0^* \oplus Y[m]$
$T \leftarrow E_K(\text{Checksum} \oplus Z[m])$ [first τ bits]
return $\mathcal{C} \leftarrow C \parallel T$

Algorithm OCB.Dec$_K$ $(N, \mathcal{C})$

Partition $\mathcal{C}$ into $C[1] \cdots C[m]\, T$
$L \leftarrow E_K(0^n)$
$R \leftarrow E_K(N \oplus L)$
for $i \leftarrow 1$ **to** m **do** $Z[i] = \gamma_i \cdot L \oplus R$
for $i \leftarrow 1$ **to** $m-1$ **do**
$\quad M[i] \leftarrow E_K^{-1}(C[i] \oplus Z[i]) \oplus Z[i]$
$X[m] \leftarrow \mathrm{len}(C[m]) \oplus L \cdot \mathtt{x}^{-1} \oplus Z[m]$
$Y[m] \leftarrow E_K(X[m])$
$M[m] \leftarrow Y[m] \oplus C[m]$
$M \leftarrow M[1] \cdots M[m]$
Checksum $\leftarrow$
$\quad M[1] \oplus \cdots \oplus M[m-1] \oplus C[m]\,0^* \oplus Y[m]$
$T' \leftarrow E_K(\text{Checksum} \oplus Z[m])$ [first τ bits]
if $T = T'$ **then return** M
$\qquad$ **else return** INVALID

Figure 1: **OCB encryption.** The message to encrypt is M and the key is K. Message M is written as $M = M[1]M[2] \cdots M[m-1]M[m]$, where $m = \max\{1, \lceil |M|/n \rceil\}$ and $|M[1]| = |M[2]| = \cdots = |M[m-1]| = n$. Nonce N is a non-repeating value selected by the party that encrypts. It is sent along with ciphertext $\mathcal{C} = C[1]C[2]C[3] \cdots C[m-1]C[m]\, T$. The Checksum is $M[1] \oplus \cdots \oplus M[m-1] \oplus C[m]\,0^* \oplus Y[m]$. Offset $Z[1] = L \oplus R$ while, for $i \geq 2$, $Z[i] = Z[i-1] \oplus L(\mathsf{ntz}(i))$. String L is defined by applying E_K to a fixed string, 0^n. For $Y[m] \oplus M[m]$ and and $Y[m] \oplus C[m]$, truncate $Y[m]$ if it is longer than the other operand. By $C[m]\,0^*$ we mean $C[m]$ padded on the right with 0-bits to get to length n. The function len represents the length of its argument as an n-bit string.

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of $\tau = 64$ is reasonable. Tags of 32 bits are standard in retail banking. Tags of 96 bits are used in IPSec. Using a tag of more than 80 bits adds questionable security benefit, though it does lengthen each ciphertext.

We let OCB-E denote the OCB mode of operation using block cipher E and an unspecified tag length. We let OCB$[E, \tau]$ denote the OCB mode of operation using block cipher E and tag length τ.

Nonces. Encryption under OCB mode requires an n-bit nonce, N. The nonce would typically be a counter (maintained by the sender) or a random value (selected by the sender). Security is maintained even if the adversary can control the nonce, subject to the constraint that no nonce may be repeated within the current session (that is, during the period of use of the current encryption key). The nonce need not be random, unpredictable, or secret.

The nonce N is needed both to encrypt and to decrypt. Typically it would be communicated, in the clear, along with the ciphertext. However, it is out-of-scope how the nonce is communicated to the party who will decrypt. In particular, we do not regard the nonce as part of the ciphertext.

Definition of the mode. See Figure 1 for a definition and illustration of OCB. The figure defines OCB encryption and decryption. The key space for OCB is the key space $\mathcal{K}$ for the underlying block cipher E.

An equivalent description. The following description may clarify what a typical implementation might do.

Key generation. Choose a random key $K \stackrel{R}{\leftarrow} \mathcal{K}$ for the block cipher. The key K is provided to both the entity that encrypts and the entity that decrypts.

Session setup. For the party that encrypts, do any key-setup associated to block-cipher enciphering. For the party that decrypts, do any key-setup associated to block-cipher enciphering and deciphering. Let $L \leftarrow E_K(0^n)$. Let m bound the maximum number of n-bit blocks that any message which will be encrypted or decrypted may have. Let $\mu \leftarrow \lceil \log_2 m \rceil$. Let $L(0) \leftarrow L$ and, for $i \in [1..\mu]$, compute $L(i) \leftarrow L(i-1) \cdot \mathtt{x}$ using a shift and a conditional xor, as described in Section 2. Compute $L(-1) \leftarrow L \cdot \mathtt{x}^{-1}$ using a shift and a conditional xor, as described in Section 2. Save the values $L(-1)$, $L(0)$, $L(1), \ldots, L(\mu)$ in a table.

Encryption. To encrypt plaintext $M \in \{0,1\}^*$ using key K nonce $N \in \{0,1\}^n$, obtaining a ciphertext $\mathcal{C}$, do the following. Let $m \leftarrow \lceil |M|/n \rceil$. If $m = 0$ then let $m \leftarrow 1$. Let $M[1], \ldots, M[m]$ be strings such that $M[1] \cdots M[m] = M$ and $|M[i]| = n$ for $i \in [1..m-1]$. Let Offset $\leftarrow E_K(N \oplus L)$. Let Checksum $\leftarrow 0^n$. For $i \leftarrow 1$ to $m-1$, do the following: let Checksum $\leftarrow$ Checksum $\oplus M[i]$; let Offset $\leftarrow$ Offset$\oplus L(\mathsf{ntz}(i))$; let $C[i] \leftarrow E_K(M[i] \oplus \text{Offset}) \oplus$Offset. Let Offset $\leftarrow$ Offset$\oplus L(\mathsf{ntz}(m))$. Let $Y[m] \leftarrow E_K(\text{len}(M[m]) \oplus L(-1) \oplus \text{Offset})$. Let $C[m] \leftarrow M[m]$ xored with the first $|M[m]|$ bits of $Y[m]$. Let Checksum $\leftarrow$ Checksum $\oplus\, Y[m] \oplus C[m]\, 0^*$. Let T be the first τ bits of $E_K(\text{Checksum} \oplus \text{Offset})$. The ciphertext is $\mathcal{C} = C[1] \cdots C[m-1] C[m]\ T$. It must be communicated along with the nonce N.

Decryption. To decrypt ciphertext $\mathcal{C} \in \{0,1\}^*$ using key K and nonce $N \in \{0,1\}^n$, obtaining a plaintext $M \in \{0,1\}^*$ or an indication Invalid, do the following. If $|\mathcal{C}| < \tau$ then return Invalid (the ciphertext has been rejected). Otherwise let C be the first $|\mathcal{C}| - \tau$ bits of $\mathcal{C}$ and let T be the remaining τ bits. Let $m \leftarrow \lceil |C|/n \rceil$. If $m = 0$ then let $m = 1$. Let $C[1], \ldots, C[m]$ be strings such that $C[1] \cdots C[m] = C$ and $|C[i]| = n$ for $i \in [1..m-1]$. Let Offset $\leftarrow E_K(N \oplus L)$. Let Checksum $\leftarrow 0^n$. For $i \leftarrow 1$ to $m-1$, do the following: let Offset $\leftarrow$ Offset $\oplus L(\mathsf{ntz}(i))$; let $M[i] \leftarrow E_K^{-1}(C[i] \oplus \text{Offset}) \oplus$Offset; let Checksum $\leftarrow$ Checksum$\oplus M[i]$. Let Offset $\leftarrow$ Offset$\oplus L(\mathsf{ntz}(m))$. Let

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$Y[m] \leftarrow E_K(\text{len}(C[m]) \oplus L(-1) \oplus \text{Offset})$. Let $M[m] \leftarrow C[m]$ xored with the first $|C[m]|$ bits of $Y[m]$. Let Checksum $\leftarrow$ Checksum $\oplus\, Y[m] \oplus C[m]\, 0^*$. Let T' be the first τ bits of $E_K(\text{Checksum} \oplus \text{Offset})$. If $T \neq T'$ then return Invalid (the ciphertext has been rejected). Otherwise, the plaintext is $M = M[1] \cdots M[m-1]M[m]$.

4 Discussion

OCB has been designed to have a variety of desirable properties. These properties are summarized in Figure 2. We now expand on some of the points made in that table, and add some further comments.

Arbitrary-length messages and no ciphertext expansion. One of the key characteristics of OCB is that any string $M \in \{0,1\}^*$ can be encrypted, and doing this yields a ciphertext $\mathcal{C}$ of length $|M| + \tau$. That is, the length of the "ciphertext core"—the portion $C = C[1] \cdots C[m]$ of the ciphertext that excludes the tag—is the same as the length of the message M. This is better, by up to n bits, than what one gets with conventional padding.

Single block-cipher key. OCB makes use of just one block-cipher key, K. While $L = E_K(0^n)$ functions rather like a key and would normally be computed at session-setup time, and while standard key-separation techniques can always be used to obtain many keys from one, the point is that, in OCB, all block-cipher invocations use the one key K. Thus only one block-cipher key needs to be setup, saving on storage space and key-setup time.

Weak nonce requirements. We believe that modes of operation that requires a random IV are error-prone. As an example, consider CBC mode, where $C[i] = E_K(M[i] \oplus C[i-1])$ and $C[0] = \text{IV}$. Many standards and many books (e.g., Schneier, *Applied Cryptography*, 2nd edition, p. 194]) suggest that the IV may be a fixed value, a counter, a timestamp, or the last block of ciphertext from the previous message. But if it is any of these things one certainly will not achieve any of the standard definitions of security [2, 15].

It is sometimes suggested that a mode which needs a random IV is preferable to one that needs a nonce: it is said that *state* is needed for a nonce, but not for making random bits. We find this argument wrong. First, a random value of sufficent length can always be used as a nonce, but a nonce can not be used as a random value. Second, the manner in which systems provide "random" IVs is invariably stateful anyway: unpredictable bits are too expensive to harvest for each IV, so one does this rarely, using state to generate pseudorandom bits from unpredictable bits harvested before. Third, the way to generate pseudorandom bits needs to use cryptography, so the prevalence of non-cryptographic pseudorandom number generators routinely results in implementation errors. Next, nonce-based schemes make it possible for the receiver to implement replay-detection with no added cryptography. Finally, nonces can be communicated using fewer bits, without any additional cryptography.

On-line. OCB encryption and decryption are "on line" in the sense that one does not need to know the length of the message in advance of encrypting or decrypting it. Instead, messages can be processed as one goes along, using constant memory, continuing until there is an indication that the message is over. An incremental interface (in the style popular for cryptographic hash functions) would be used to support this functionality.

Significance of being efficient. Shaving off a few block-cipher calls or a few bytes of ciphertext may not seem important. But often one is dealing with short messages; for example, roughly a

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Security Function	**Authenticated encryption**. Provides both privacy and authenticity, eliminating the need to compute a separate MAC. Specifically, the scheme achieves authenticity of ciphertexts [6, 7, 21] and indistinguishability under chosen-plaintext attack [2, 15].
Error Propagation	**Infinite**. If the ciphertext is corrupted in any manner then the received ciphertext will almost certainly (probability $\approx 1 - 2^{-\tau}$) be rejected.
Synchronization	**Optional**. If the nonce N is transmitted along with each ciphertext, there are no synchronization requirements. If it is not sent (to save transmission bits) the receiver must maintain the corresponding value.
Parallelizability	**Fully parallelizable**. Both encryption and decryption are fully parallelizable: all block-cipher invocations (except the first and last) may be computed at the same time.
Keying Material	**One block-cipher key**. One needs a single key, K, which keys all invocations of the underlying block cipher.
Ctr/IV/Nonce Requirements	**Single-use nonce**. The encrypting party must supply a new nonce with each message it encrypts. The nonce need not be unpredictable or secret. The nonce is n bits long (but it would typically be communicated using fewer bits, as determined by the application).
Memory Requirements	**Very modest**. About $6n$ bits beyond the key are sufficient for internal calculations. Implementations may choose whether or not to store $L(i)$-values, allowing some tradeoff between memory and simplicity/speed.
Pre-processing Capability	**Limited.** During key-setup the string L would typically be precomputed (one block cipher call), as would the first few $L(i)$ values, and maybe $L \cdot \mathtt{x}^{-1}$. The block-cipher key K would be converted into its convenient representation. Unlike counter mode, additional precomputation prior to knowing the string to encrypt/decrypt is not possible.
Message-Length Requirements	**Any bit string allowed.** Any string $M \in \{0,1\}^*$ may be encrypted, including the empty string and strings which are not an integral number of bytes. The length of the string does need not be known in advance.
Ciphertext Expansion	**Minimal possible** (for a scheme meeting the desired privacy notion). Expansion is 0–n bits for the tag plus 0–n bits for the nonce. The former depends on a user-specified parameter τ, with 32–80 bits being typical. Messages which are not a multiple of the blocksize do not receive additional expansion due to padding.
Other Characteristics	**Efficiency**: Uses $\lceil \lvert M \rvert / n \rceil + 2$ block-cipher calls and very efficient offset-calculations. **Endian neutrality**: Can be implemented equally efficiently on big-endian and little-endian machines. **Provable security**: The mode provably meets its goals, assuming the underlying block cipher meets now-standard cryptographic assumptions.

Figure 2: **Summary properties of OCB.**

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third of the messages on the Internet backbone are 43 bytes. If one is encrypting messages of such short lengths, one should be careful about message expansion and extra computational work since, by percentage, the inefficiencies can be large.

The argument has been made that making a major effort to save a factor of two in computational efficiency is marginal in the first place: "Moore's law" will soon deliver such an improvement anyway, by way of faster hardware. We are not persuaded. Concommitent with processors getting faster has been security becoming increasingly at issue, and low-power processors becoming all the more prevalent. The result is a need to cryptographically process more and more data, and often by "dumb" execution vehicles that have plenty of other things to do. Hardware advances have changed our understanding of what efficiency entails but, to date, hardware advances have not made cryptographic efficiency any less important.

Endian neutrality. In contrast to a scheme based on mod p arithmetic (for p a prime just less than 2^n) or mod 2^n arithmetic, there is almost no endian-favoritism implicit in the definition of OCB. (The exception is that, because of our use of standard mathematical conventions, the left shift used for forming $L(i+1)$ from $L(i)$ is more convenient under a big-endian convention, as is the right shift used for forming $L(-1) = L \cdot \mathtt{x}^{-1}$ from L.)

Optional pre-processing. Implementations can choose how many $L(i)$ values to precompute. As only one block-cipher call is involved, plus some shifts and conditional xors, it is feasible to do no preprocessing; OCB-AES is appropriate even when each session is a single, short message.

Provable security. Provable security has become a popular goal for practical protocols. This is because it provides the best way to gain assurance that a cryptographic scheme does what it should. For a scheme which enjoys provable security one does not need to consider attacks, since successful ones imply successful attacks on some simpler object.

When we say that "OCB is provably secure" we are asserting the existence of two theorems. One says that *if* an adversary A could do a good job at forging ciphertexts with OCB$[E,\tau]$ (the adversary does this much more than a $2^{-\tau}$ fraction of the time) then there would be *another* adversary B that does a good job at distinguishing $(E_K(\cdot), E_K^{-1}(\cdot))$, for a random key K, from $(\pi(\cdot), \pi^{-1}(\cdot))$, for a random permutation $\pi \in \mathrm{Perm}(n)$. The other theorem says that *if* an adversary A could do a good job at distinguishing OCB$[E,\tau]$-encrypted messages from random strings, then there would be *another* adversary B that does a good job at distinguishing $E_K(\cdot)$, for a random key K, from $\pi(\cdot)$, for a random permutation $\pi \in \mathrm{Perm}(n)$. Theorems of this sort are called *reductions.* In cryptography, provable security means giving reductions (along with the associated definitions).

Provable security begins with Goldwasser and Micali [15], though the style of provable security which we use here—where the primitive is a block cipher, the scheme is a usage mode, and the analysis is concrete (no asymptotics)—is the approach of Bellare and Rogaway [2, 4, 5].

It is not enough to know that there is some sort of provable-security result; one should also understand the definitions and the bounds. We have already sketched the definitions. When we speak of the bounds we are addressing "how effective is the adversary B in terms of the efficacy of adversary A" (where A and B are as above). For OCB, the bounds can be roughly summarized as follows. An adversary can always forge with probability $1/2^\tau$. Beyond this, the maximal added advantage is at most $\sigma^2/2^n$, where σ is the total number of blocks the adversary sees. The privacy bound likewise degrades as $\sigma^2/2^n$. The conclusion is that one is safe using OCB as long as the underlying block cipher is secure and σ is small compared to $2^{n/2}$. This is the same security degradation one observes for CBC encryption and in the bound for the CBC MAC [2, 5]. This kind of security loss was the main motivation for choosing a block length for AES of $n = 128$ bits.

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Comparison with Jutla's bound. More precisely, but still ignoring lower-order terms, our privacy and authenticity bounds are $1.5\,\sigma^2/2^n$, while Jutla's authenticity bound [19] is insignificantly worse at $2\,\sigma^2/2^n$ and his privacy bound, rescaled to $[0,1]$, looks insignificantly worse at $3\,\sigma^2/2^n$. Magnifying the latter difference is that the privacy results assume different defintions. Jutla adopts the find-then-guess definition of privacy [2, 15], while we use an indistinguishability-from-random-bits definition. The former captures an adversary's inability to distinguish ciphertexts corresponding to a pair adversarilly-selected, equal-length messages. The latter captures an adversary's inability to distinguish a ciphertext from a random string of the same length. Indistinguishability-from-random-bits implies find-then-guess security, and by a tight reduction, but find-then-guess secure does not imply indistinguishability-from-random-bits. Still, Jutla's scheme probably satisfies the stronger definition.

Simplicity. Simplicity has been a central design goal. Some of OCB's characteristics that contribute to simplicity are:

- Short and full final-message-blocks are handled without making a special case: the treatment of all messages is uniform, regardless of their length.
- Only the simplest form of padding is used: append a minimal number of 0-bits to make a string whose length is a multiple of n. This method is computationally fastest and helps avoid a proliferation of cases in the analysis.
- Only one algebraic structure is used throughout the algorithm: the finite field $\mathrm{GF}(2^n)$.
- In forming the sequence of offsets, Gray-code coefficients are taken monotonically, starting at 1 and stopping at m. One never goes back to some earlier offset, uses a peculiar starting point, or forms more offsets than there are blocks.

Not fixing how the nonce is communicated. We do not specify how the nonce is chosen or communicated. Formally, it is not part of the ciphertext (though the receiving party needs it to decrypt). In many contexts, there is already a natural value to use as a nonce (e.g., a sequence number already present in a protocol flow, or implicit because the parties are communicating over a reliable channel). Even when a protocol is designed from scratch, the number of bits needed to communicate the nonce will vary. In some applications, 32 or 8 bits is enough. For example, one might have reason to believe that there are at most 2^{32} messages that will flow during the connection, or one may communicate only the lowest 8 bits of a sequence number, counting on the Receiver to anticipate the high-order bits.

Not fixing the tag length. The number of bits necessary for the tag vary according to the application. In a context where the adversary obtains something quite valuable from a successful forgery, one may wish to choose a tag length of 80 bits or more. In contexts such as authenticating a video stream, where an adversary would have to forge a significant fraction of the frames even to have a noticeable effect on the image, an 8-bit tag may be appropriate. With no universally correct value to choose, it is best to leave this parameter unspecified.

Short tags seem to be more appropriate for OCB than for some other MACs, particularly Carter-Wegman MACs. Many Carter-Wegman MACs have the property that if you can forge one message with probability δ then you can forge an arbitrary set of (all correct) messages with probability δ. This does not appear to be true for OCB (though we have not investigated formalizing or proving such properties).

Forming R using a block-cipher call. During our work we discovered that there are methods for authenticated-encryption which encrypt M using $\lceil |M|/n \rceil + 1$ block-cipher calls, as opposed to

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our $\lceil |M|/n \rceil + 2$ calls. Shai Halevi has also made this finding [16]. However, the methods we know to shave off a block-cipher call either require an unpredictable IV instead of a nonce, or they add conceptual and computational complexity to compute the initial offset R by non-cryptographic means (e.g., using a finite-field multiplication of the nonce and a key variant).

AVOIDING MOD 2^n ADDITION. Our earlier designs included a scheme based on modular 2^n addition ("addition" for the remainder of this paragraph). Basing an authenticated-encryption scheme on addition is an interesting idea due to Gligor and Donescu [14]. Compared to our GF(2^n)-based approach ("xor" for the remainder of this paragraph), an addition-based scheme is quicker to understand a specification for, and may be easier to implement. But the use of addition (where $n \geq 128$) has several disadvantages:

- The bit-asymmetry of the addition operator implies that the resulting scheme will have a bias towards big-endian architectures or little-endian architectures; there will be no way to achieve an endian-neutral scheme. The AES algorithm was constructed to be endian-neutral and we wanted OCB-AES to share this attribute.
- Modular addition of n-bit words is unpleasant for implementations using high-level languages, where one normally has no access to any add-with-carry instruction.
- Modular addition of n-bit words is not parallelizable. As a consequence, dedicated hardware will perform this operation more slowly than xor, and, correspondingly, modern processors can xor two n-bit quantities faster than they can add them.
- The concrete security bound appears to be worse (though still not bad) with an addition-based scheme: the degradation would seem to be $\Theta(\lg \bar{m})$, where $\bar{m}$ is the maximal message length.

We eventually came to feel that the simplicity benefit of addition-based schemes was not quite real: these schemes seem harder to understand, to prove correct, and to implement well.

LAZY MOD p ADDITION. Let p be the largest prime less than 2^n. An earlier design [30] allowed one to produce offset $Z[i]$ from $Z[i-1]$ by adding L to $Z[i-1]$, mod 2^n, and then adding $\delta = 2^n - p$ whenever the first addition generated a carry. Now $X[m]$ would be defined by $\text{len}(M[m]) \oplus \overline{Z[m]}$, say, where $\overline{Z[m]}$ is the bitwise complement of $Z[m]$. It appears that, unlike a mod 2^n scheme, xors can still be used to combine offsets with message blocks and enciphered message blocks. This might make an xor-based lazy-mod-p approach more attractive than a mod-2^n approach. But in order to propagate a single scheme, avoid endian favoritism, and avoid complicating an already complex proof, and we chose not to propagate lazy-mod-p-addition.

DEFINITION OF THE CHECKSUM. An initially odd-looking aspect of OCB's definition is the definition of Checksum $= M[1] \oplus \cdots M[m-1] \oplus C[m]\, 0^* \oplus Y[m]$. In Jutla's scheme, where one assumes that all messages are a positive multiple of the block length, the checksum is the simpler-looking $M[1] \oplus \cdots M[m-1] \oplus M[m]$. We comment that these two definitions are identical in the case that $|M[m]| = n$. What is more, the definition Checksum $= M[1] \oplus \cdots M[m-1] \oplus M[m]\, 0^*$ turns out to be the wrong way to generalize the Checksum to allow for short-final-block messages; in particular, the scheme using that checksum is easily attacked.

AVOIDING PRETAG COLLISIONS. Many of our earlier schemes, including [30], allowed the adversary to force a "pretag collision." Recall that we compute the tag T by computing a "pretag" $X[m+1] =$ Checksum $\oplus$ SomeOffset, forming a value $Y[m+1] = E_K(X[m+1])$, and then forming T by doing further processing to $Y[m+1]$. For a scheme of this form, we say that an adversary can force a pretag collision if there is an N, $\bar{M}$ that can be encrypted, getting $\bar{C}\, \bar{T}$, and then a forgery attempt N, $C\, T$ can be made such that, in it, the pretag $X[m+1]$ will coincide with a value $X[i]$ or $\bar{X}[i]$

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at which the block cipher E was already evaluated.

We designed OCB so that an adversary can not force pretag collisions. The presence of pretag collisions substantially complicates proofs, since one can not follow a line of argument that shows that tags are unpredictable because each pretag-value is almost certainly new. For schemes like IAPM, where pretag collisions arise, this intuition is simply wrong. Beyond this, note that in the presence of pretag collisions one must modify $Y[m+1]$ by an amount Δ that depends on at least the key and nonce. Say that the modification is by xor, and one wants to be able to pull off an arbitrary bit as a 1-bit authentication tag. Then every bit of Δ will have to be adversarially unpredictable,. This is unfortunate, as many natural ways to make Δ fail to have this property. Suppose, for example, the first couple bits of L are forced to zero, as suggested by [30], and $\Delta = L \cdot (m+1)$. Then, for small m, the first bit of Δ will be zero. This can be exploited to give an attack on the xor-based scheme of [30] when $\tau = 1$. Similarly, for i a power of two, $\Delta = iL \bmod 2^n$ ends in a 0-bit, so had [30] taken the tag to be the last τ bits instead of the first τ bits, one would again have an attack on 1-bit tags. A scheme would be arcane, at best, if certain bits of the full tag are usable and other bits are not.

Block-cipher circuit-depth. One efficiency measure we have not discussed is the circuit depth of an encryption scheme as measured in terms of block-cipher gates. For OCB encryption, this number is three: a call to form R; calls to form the ciphertext core; and a call to compute the tag. Block-cipher circuit-depth serves as a lower bound for latency in an agressively parallel environment. Reducing the block-cipher circuit-depth to one or two is possible, but the benefit does not seem worth the associated drawbacks.

5 Theorems

5.1 Security Definitions

We begin with the requisite definitions. These are not completely standard because OCB uses a nonce, and we wish to give the adversary every possible advantage (more than is available in real life) by allowing her to choose this nonce (though we forbid the adversary from choosing the same nonce twice).

Syntax. We extend the syntax of an encryption scheme as given in [2]. A (nonce-using, symmetric) encryption scheme Π is a triple $\Pi = (\mathcal{K}, \mathcal{E}, \mathcal{D})$ and an associated number n (the nonce length). Here $\mathcal{K}$ is a finite set and $\mathcal{E}$ and $\mathcal{D}$ are deterministic algorithms. Encryption algorithm $\mathcal{E}$ takes strings $K \in \mathcal{K}$, $N \in \{0,1\}^n$, and $M \in \{0,1\}^*$, and returns a string $\mathcal{C} \leftarrow \mathcal{E}_K(N, M)$. Decryption algorithm $\mathcal{D}$ takes strings $K \in \mathcal{K}$, $N \in \{0,1\}^n$, and $\mathcal{C} \in \{0,1\}^*$, and returns $\mathcal{D}_K(N, M)$, which is either a string $M \in \{0,1\}^*$ or the distinguished symbol Invalid. If $\mathcal{C} \leftarrow \mathcal{E}_K(N, M)$ then $\mathcal{D}_K(N, \mathcal{C}) = M$.

Privacy. We give a particularly strong definition of privacy, one asserting indistinguishability from random strings. This notion is easily seen to imply more standard definitions [2], and by tight reductions. Consider an adversary A who has one of two types of oracles: a "real" encryption oracle or a "fake" encryption oracle. A real encryption oracle, $\mathcal{E}_K(\cdot,\cdot)$, takes as input N, M and returns $\mathcal{C} \leftarrow \mathcal{E}_K(N, M)$. It is assumed that $|\mathcal{C}| = \ell(|M|)$ depends only on $|M|$. A fake encryption oracle, $\$(\cdot,\cdot)$, takes as input N, M and returns a random string $\mathcal{C} \stackrel{R}{\leftarrow} \{0,1\}^{\ell(|M|)}$. Given adversary A and encryption scheme $\Pi = (\mathcal{K}, \mathcal{E}, \mathcal{D})$, define $\mathbf{Adv}^{\mathrm{priv}}_{\Pi}(A) = \Pr[K \stackrel{R}{\leftarrow} \mathcal{K} :\ A^{\mathcal{E}_K(\cdot,\cdot)} = 1] - \Pr[K \stackrel{R}{\leftarrow} \mathcal{K} :\ A^{\$(\cdot,\cdot)} = 1]$.

An adversary A is *nonce-respecting* if it never repeats a nonce: if A asks its oracle a query

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(N, M) it will never subsequently ask its oracle a query (N, M'), regardless of its coins (if any) and regardless of oracle responses. All adversaries are assumed to be nonce-respecting.

Authenticity. We extend the notion of integrity of ciphertexts of [6, 7, 21]. Fix an encryption scheme $\Pi = (\mathcal{K}, \mathcal{E}, \mathcal{D})$ and run an adversary A with an oracle $\mathcal{E}_K(\cdot,\cdot)$ for some key K. Adversary A *forges* (in this run) if A is nonce-respecting, A outputs $(N, \mathcal{C})$ where $\mathcal{D}_K(N, \mathcal{C}) \neq$ Invalid, and A made no earlier query (N, M) which resulted in a response $\mathcal{C}$. Let $\mathbf{Adv}^{\mathrm{auth}}_{\Pi}(A) = \Pr[K \stackrel{R}{\leftarrow} \mathcal{K} :\ A^{\mathcal{E}_K(\cdot,\cdot)} \text{ forges}\ ]$. We stress that the nonce used in the forgery attempt may coincide with a nonce used in one of the adversary's queries.

Block ciphers and PRFs. A *function family* from n-bits to n-bits is a map $E : \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$ where $\mathcal{K}$ is a finite set of strings. It is a *block cipher* if each $E_K(\cdot) = E(K, \cdot)$ is a permutation. Let $\mathrm{Rand}(n)$ denote the set of all functions from $\{0,1\}^n$ to $\{0,1\}^n$ and let $\mathrm{Perm}(n)$ denote the set of all permutations from $\{0,1\}^n$ to $\{0,1\}^n$. These sets can be regarded as function families by imagining that each member is specified by a string. Letting $E_K^{-1}(Y)$ be the unique string X such that $E_K(X) = Y$, define

$$\begin{aligned}
\mathbf{Adv}^{\mathrm{prf}}_{E}(A) &= \Pr[K \stackrel{R}{\leftarrow} \mathcal{K} :\ A^{E_K(\cdot)} = 1] - \Pr[\rho \stackrel{R}{\leftarrow} \mathrm{Rand}(n) :\ A^{\rho(\cdot)} = 1] \\
\mathbf{Adv}^{\mathrm{prp}}_{E}(A) &= \Pr[K \stackrel{R}{\leftarrow} \mathcal{K} :\ A^{E_K(\cdot)} = 1] - \Pr[\pi \stackrel{R}{\leftarrow} \mathrm{Perm}(n) :\ A^{\pi(\cdot)} = 1] \\
\mathbf{Adv}^{\mathrm{sprp}}_{E}(A) &= \Pr[K \stackrel{R}{\leftarrow} \mathcal{K} :\ A^{E_K(\cdot), E_K^{-1}(\cdot)} = 1] - \Pr[\pi \stackrel{R}{\leftarrow} \mathrm{Perm}(n) :\ A^{\pi(\cdot), \pi^{-1}(\cdot)} = 1]
\end{aligned}$$

5.2 Theorem Statements

We give information-theoretic bounds on the authenticity and the privacy of OCB. Proofs are in Appendix B.

Theorem 1 [Authenticity] *Fix OCB parameters n and τ. Let A be an adversary that asks q queries and then makes its forgery attempt. Suppose the q queries have aggregate length of σ blocks, and the adversary's forgery attempt has at most c blocks. Let $\bar{\sigma} = \sigma + 2q + 5c + 11$. Then*

$$\mathbf{Adv}^{\mathrm{auth}}_{\mathrm{OCB}[\mathrm{Perm}(n),\tau]}(A) \ \leq\ \frac{1.5\,\bar{\sigma}^2}{2^n} + \frac{1}{2^\tau}$$

The aggregate length of queries $M_1, \ldots, M_q$ means the number $\sigma = \sum_{r=1}^{q} \|M_r\|_n$.

It is standard to pass to a complexity-theoretic analog of Theorem 1, but in doing this one will need access to an E^{-1} oracle in order to verify a forgery attempt, which translates into needing the strong PRP assumption. One gets the following. Fix OCB parameters n and τ, and a block cipher $E : \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$. Let A be an adversary that asks q queries and then makes its forgery attempt. Suppose the q queries have aggregate length of σ blocks, and the adversary's forgery attempt has at most c blocks. Let $\bar{\sigma} = \sigma + 2q + 5c + 11$. Let $\delta = \mathbf{Adv}^{\mathrm{auth}}_{\mathrm{OCB}[E,\tau]}(A) - 1.5\,\bar{\sigma}^2/2^n - 1/2^\tau$. Then there is an adversary B for attacking block cipher E that achieves advantage $\mathbf{Adv}^{\mathrm{sprp}}_{E}(B) \geq \delta$. Adversary B asks at most $q' = \sigma + 2q + 5c + 11$ oracle queries and has a running time which is equal to A's running time plus the time to compute E or E^{-1} at q' points plus additional time which is $\alpha n \bar{\sigma}$, where the constant α depends only on details of the model of computation.

The privacy of OCB is given by the following result.

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Theorem 2 [Privacy] *Fix OCB parameters n and τ. Let A be an adversary that asks q queries, these having aggregate length of σ blocks. Let $\bar{\sigma} = \sigma + 2q + 3$. Then*

$$\mathbf{Adv}^{\mathrm{priv}}_{\mathrm{OCB}[\mathrm{Perm}(n),\tau]}(A) \quad \leq \quad \frac{1.5\,\bar{\sigma}^2}{2^n}$$

It is standard to pass to a complexity-theoretic analog of Theorem 2. One gets the following. Fix OCB parameters n and τ, and a block cipher $E : \mathcal{K} \times \{0,1\}^n \rightarrow \{0,1\}^n$. Let A be an adversary that asks q queries, these having aggregate length of σ blocks. Let $\bar{\sigma} = \sigma + 2q + 3$. Let $\delta = \mathbf{Adv}^{\mathrm{auth}}_{\mathrm{OCB}[E,\tau]}(A) - 1.5\,\bar{\sigma}^2/2^n$. Then there is an adversary B for attacking block cipher E that achieves advantage $\mathbf{Adv}^{\mathrm{prp}}_{E}(B) \geq \delta$. Adversary B asks at most $q' = \sigma + 2q + 1$ oracle queries and has a running time which is equal to A's running time plus the time to compute E at q' points plus additional time which is $\alpha n \bar{\sigma}$, where the constant α depends only on details of the model of computation.

6 Performance

Abstract accounting. OCB uses $\lceil |M|/n \rceil + 2$ block-cipher calls to encrypt a nonempty message M. (The empty string takes three block-cipher calls.) We compare this with CBC encryption and CBC encryption plus a CBC MAC:

- "Basic" CBC encryption, where one assumes a random IV and a message which is a multiple of the block length, uses two fewer block-cipher calls—a total of $|M|/n$.
- A more fair comparison sets IV $= E_K(N)$ for CBC encryption (so both schemes can use a not-necessarily-random nonce), and uses obligatory 10^* padding (so both schemes can handle arbitrary strings). This would bring the total for CBC to $\lceil (|M| + 1)/n \rceil + 1$ block-cipher calls, coinciding with OCB when $|M|$ is a multiple of the block length, and using one fewer block-cipher call otherwise.
- If one combines the basic CBC encryption with a MAC, say MACing the ciphertext, then the CBC-encryption will use a number of block-cipher calls as just discussed, while the CBC MAC will use between $\lceil |M|/n \rceil + 1$ and $\lceil (|M|+1)/n \rceil + 3$ block-cipher calls, depending on padding conventions and the optional processing done to the final block in order to ensure security across messages of varying lengths. So the total will be as few as $2\lceil |M|/n \rceil + 1$ or as many as $2\lceil (|M|+1)/n \rceil + 4$ block-cipher calls. Thus OCB saves between $\lceil |M|/n \rceil - 1$ and $\lceil |M|/n \rceil + 3$ block-cipher calls compared to separate CBC encryption and CBC MAC computation

As with any mode, there is overhead beyond the block-cipher calls. Per block, this overhead is about four n-bit xor operations, plus associated logic. The work for this associated logic will vary according to whether or not one precomputed $L(i)$-values and many additional details.

Though some of the needed $L(i)$-values are likely to be precomputed, computing all of them "on the fly" is not inefficient. Starting with 0^n we form successive offsets by xoring the previous offset with L, $2 \cdot L$, L, $4 \cdot L$, L, $2 \cdot L$, L, $8 \cdot L$, and so forth. So half the time we use L itself; a quarter of the time we use $2 \cdot L$; one eighth of the time we use $4 \cdot L$; and so forth. Thus the expected number of times to multiply by $\mathtt{x}$ in order to compute an offset is at most $\sum_{i=1}^{\infty} i/2^{i+1} = 1$. Each $a \cdot \mathtt{x}$ instruction requires an n-bit xor and a conditional 32-bit xor. Said differently, for any $m > 0$, the total number of $a \cdot \mathtt{x}$ operations needed to compute $\gamma_1 \cdot L$, $\gamma_2 \cdot L$, $\ldots$, $\gamma_m \cdot L$ is $\sum_{i=1}^{m} \mathsf{ntz}(i)$, which is less than m. The above assumed that one does not retain or precompute any $L(i)$ value beyond $L = L(0)$. Suppose that one precomputes $L(-1)$, $L(0)$, $L(1)$, $L(2)$, $L(3)$. Computing and

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Algorithm	64 B	256 B	1 KB	4 KB
OCB encrypt	24.7 (395)	18.5 (296)	**16.9** (271)	16.7 (267)
ECB encrypt	15.1 (241)	15.0 (239)	**14.9** (238)	14.9 (238)
CBC encrypt	15.9 (254)	15.9 (254)	**15.9** (255)	15.9 (256)
CBC mac	19.2 (307)	16.3 (261)	**15.5** (248)	15.3 (246)

Figure 3: **Performance results** from Lipmaa [24], in cycles per byte (and cycles per 16-byte block) on a Pentium III. The block cipher is AES128. Code is written in assembly.

storing the four values beyond $L = L(0)$ is cheaper than computing L itself, which required an application of E_K. But now, in forming offsets, the desired multiple of L will have be available at least $1/2 + 1/4 + 1/8 + 1/16 \approx 94\%$ of the time. When it has not been precomputed it must be calculated, starting from $L(3)$, so the amortized number of multiplications by $\mathtt{x}$ has been reduced to $\sum_{i=1}^{\infty} = i/2^{i+4} = 0.125$.

Experimental results. In Table 3 we report, with permission, some experimental results by Helger Lipmaa [24]. On a Pentium III, in optimized assembly, Lipmaa implemented OCB encryption, ECB encryption, CBC encryption, and the CBC MAC. The last three modes were implemented in their “raw” forms, where one does no padding and assumes that the message acted on is a positive multiple of the block length. For CBC encryption, the IV is fixed. The underlying block cipher is AES128.

Focusing on messages of 1 KByte, OCB incurs about 6.4% overhead compared to CBC encryption, and that the algorithm takes about 54% of the time of a CBC encryption + CBC MAC. Lipmaa points out that overhead is so low that, in his experiments, an assembly AES128 with a C-code CBC-wrapper is slightly slower than the same AES128 with an assembly OCB-wrapper. Lipmaa’s (size-unoptimized) code is 7.2 KBytes, which includes unrolling an (already unrolled) AES128 implementation (2.2 KBytes) three times.

Some aspects of the experiments above are unfavorable to OCB, making the performance estimates conservative. In particular, the “raw” CBC MAC needs to be modified to correctly handle length-variability, and doing so is normally done in a way that results in additional block-cipher calls. And when combined with CBC encryption, the CBC MAC should be taken over the full ciphertext, including the nonce, which would add an extra block-cipher call. Finally, an extra block-cipher call would normally be performed by CBC to correctly compute the IV from a nonce.

The results above are for a serial execution environment. With plenty of registers and multiple instruction pipes, OCB, properly implemented, will be faster than CBC.

Acknowledgments

At CRYPTO ’00, Virgil Gligor described [14] to Rogaway, Charanjit Jutla gave a rump-session talk on [20], and Elaine Barker announced a first modes-of-operation workshop organized by NIST. These events inspired [30], which evolved into the current work. After the first workshop NIST made a second call for proposals, and OCB took its final form in response to this call [31]. We appreciate NIST’s effort to solicit and evaluate modern modes of operation. Elaine Barker, Morris Dworkin, and Jim Foti are among those involved.

We received useful feedback from Michael Amling, Paulo Barreto, Johan Håstad, Hugo Krawczyk, Helger Lipmaa, David McGrew, Silvio Micali, Ilya Mironov, Alberto Pascual, Tom Shrimpton, and

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David Wagner. Special thanks to Michael and Ilya for their careful proofreading, and Helger for doing a state-of-the-art assembly implementation, along with providing associated timing data. Thanks to the anonymous CCS referees for their comments.

This work was carried out while Rogaway was on leave of absence from UC Davis, visiting the Department of Computer Science, Faculty of Science, Chiang Mai University. No external funding has been used for this research, but the submitter gratefully acknowledges a recent gift by Cisco Systems. Many thanks to Cisco for their support.

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A Brief History

An April 1999 paper by Gligor and Donescu describes an authenticated-encryption scheme they call PCBC [10]. The mode is wrong, as pointed out by Jutla [17]. That paper gives the first apparently correct schemes: IACBC and IAPM. Shortly after Jutla's paper appeared, Gligor and Donescu described a different scheme, XCBC [11], which is similar to IACBC. The most conspicuous difference between XCBC and IACBC is the former's use of mod 2^n addition for forming offsets and adding them in. (In contrast, IACBC makes offsets using either xor or mod p addition, for p a prime just less than 2^n.)

A first call by NIST for modes of operation brought contributions [12, 18] based on [11, 17] and a contribution by Rogaway [30] that built on [17]. In [18], Jutla began to employ a Gray-code ordering for combining basis offsets, a refinement independently introduced in [30], along with several further tricks to improve offset production, to use a single block-cipher key, and to extend the domain to $\{0,1\}^*$ while ensuring that the ciphertext core (the ciphertext without the tag) has the same length as the plaintext.

A second call by NIST resulted in [13, 19, 31], which were revisions to [12, 18, 30], respectively. In [19], Jutla now emphasized IAPM, and he adopted lazy mod-p addition for making offsets, first described in [30]. In [13], Gligor and Donescu now describe four authenticated-encryption modes, one of which, XECBS-XOR, is parallelizable. The modes incorporate some features introduced in [30] to deal with messages of arbitrary length and to use a single key. (The techniques are pushed less far and, in particular, there is ciphertext expansion when plaintexts are not a multiple of the block size.) In [31], Rogaway et al settled on one particular mechanism to make offsets (three are described in [30]), and made further refinements to [30].

Briefly comparing OCB and IAPM [19], the latter uses two separate keys and is defined only for messages which are a multiple of the block length. Once a padding regime is included, say obligatory 10^* padding, ciphertexts will be longer than OCB's by 1 to n bits. IAPM supports offset-production using either lazy mod-p addition or an xor-based scheme. The latter is not competitive with OCB in terms of session-setup costs.

The initial version of Jutla's work [17] claimed a proof, and included ideas towards one. A subsequent writeup by Halevi [16] was more rigorous.

Patents. The summary above ignores associated patent applications. Gligor/VDG, Jutla/IBM, and Rogaway have all indicated that there were such filings. All parties have provided statements to NIST promising reasonable and nondiscriminatory licensing.

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Definitions. Though the fast authenticated-encryption goal is folklore, provable-security treatments are recent. The first definition for authenticated encryption is due to Bellare and Rogaway [7] and, independently, Katz and Yung [21]. Bellare and Namprempre were the first to seriously investigate the properties of authenticated-encryption and the generic-composition paradigm [6].

B Proofs

B.1 Structure of the Proofs

Our proof of Theorem 1 is based on three lemmas. The first, the *structure lemma*, relates the authenticity of OCB to three functions: the M-collision probability, denoted $\mathrm{Mcoll}_n(\cdot)$, the MM-collision probability, denoted $\mathrm{MMcoll}_n(\cdot,\cdot)$, and the CM-collision probability, denoted $\mathrm{CMcoll}_n(\cdot,\cdot)$. We state this lemma and then explain its purpose and the functions to which it refers.

Lemma 1 **[Structure lemma]** *Fix OCB parameters n and τ. Let A be an adversary that asks q queries and then makes its forgery attempt. Suppose the q queries have aggregate length of σ blocks, and the adversary's forgery attempt has at most c blocks. Let $\bar{\sigma} = \sigma + 2q + 5c + 11$. Let $\mathrm{Mcoll}_n(\cdot)$, $\mathrm{MMcoll}_n(\cdot,\cdot)$ and $\mathrm{CMcoll}_n(\cdot,\cdot)$ be the M-, MM-, and CM-collision probabilities. Then*

$$\mathbf{Adv}^{\mathrm{auth}}_{\mathrm{OCB}[\mathrm{Perm}(n),\tau]}(A) \le \max_{\substack{m_1,\ldots,m_q \\ \sum m_i = \sigma \\ m_i \ge 1}} \left\{ \sum_{r\in[1..q]} \mathrm{Mcoll}_n(m_r) + \sum_{1\le r<s\le q} \mathrm{MMcoll}_n(m_r, m_s) + \sum_{r\in[1..q]} \mathrm{CMcoll}_n(c, m_r) \right\} + \frac{\bar{\sigma}^2}{2^{n+1}} + \frac{1}{2^\tau}$$

What this lemma does. The structure lemma provides a recipe for measuring the maximal forging probability of an adversary attacking the authenticity of OCB: compute the M-, MM- and CM- collision probabilities, and then put them together using the formula of the lemma.

Informally, $\mathrm{Mcoll}_n(m)$ measures the probability of running into trouble when the adversary asks a single query of the specified length. Trouble means the occurrence of any collision in the associated block-cipher-input values. This includes the "special" input 0^n (used to define $L = E_K(0^n)$ and $N \oplus L$ (used to define $R = E_K(N \oplus L)$). Informally, $\mathrm{MMcoll}_n(m, \bar{m})$ measures the probability of running into trouble when the adversary asks some two particular oracle queries of the specified lengths. Trouble means that a block-cipher input associated to the first message coincides with a block-cipher input associated to the second message. Informally, $\mathrm{CMcoll}_n(c, \bar{m})$ measures the probability of running into trouble when the adversary tries to forge some particular ciphertext C of the specified block length c, there having been an earlier query of some particular message M of the specified block length m, it receiving some particular response. This time trouble basically refers to the final block-cipher input for the forgery attempt, $X[c+1]$, coinciding with some earlier block-cipher input.

The structure lemma simplifies the analysis of OCB in two ways. First, it allows one to excise adaptivity as a concern. Dealing with adaptivity is a major complicating factor in proofs of this type. Second, it allows one to concentrate on what happens to fixed pairs of messages. It is easier to think about what happens with two messages than what is happening with all $q+1$ of them.

The M- and MM-collision probability. We next define the M-collision probability and the MM-collision probability, and then state our upper bound on these functions.

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Definition 1 **[M- and MM-collision probabilities]** *Fix n and let $M = M[0] \cdots M[m+1]$ and $\bar{M} = \bar{M}[0] \cdots \bar{M}[\bar{m}+1]$ be strings of at least $2n$ bits, where each $M[i]$ and $\bar{M}[j]$ has n bits. Choose $L, R, \bar{R} \xleftarrow{R} \{0,1\}^n$ and then associate to M and $\bar{M}$ the points*

$$
\begin{array}{llll}
X[-1] & = 0^n & & \\
 & & & \\
X[0] & = M[0] \oplus L & \bar{X}[0] & = \bar{M}[0] \oplus L \\
X[1] & = M[1] \oplus \gamma_1 \cdot L \oplus R & \bar{X}[1] & = \bar{M}[1] \oplus \gamma_1 \cdot L \oplus \bar{R} \\
X[2] & = M[2] \oplus \gamma_2 \cdot L \oplus R & \bar{X}[2] & = \bar{M}[2] \oplus \gamma_2 \cdot L \oplus \bar{R} \\
\vdots & & \vdots & \\
X[m-1] & = M[m-1] \oplus \gamma_{m-1} \cdot L \oplus R & \bar{X}[\bar{m}-1] & = \bar{M}[\bar{m}-1] \oplus \gamma_{\bar{m}-1} \cdot L \oplus \bar{R} \\
X[m] & = M[m] \oplus (\gamma_m \oplus \mathit{huge}) \cdot L \oplus R & \bar{X}[\bar{m}] & = \bar{M}[\bar{m}] \oplus (\gamma_{\bar{m}} \oplus \mathit{huge}) \cdot L \oplus \bar{R} \\
X[m+1] & = M[m+1] \oplus \gamma_{\bar{m}} \cdot L \oplus R & \bar{X}[\bar{m}+1] & = \bar{M}[\bar{m}+1] \oplus \gamma_m \cdot L \oplus \bar{R}
\end{array}
$$

and the multisets

$$
\begin{array}{lcl}
\mathcal{X}_0 & = & \{\, X[-1],\ X[0],\ X[1],\ \ldots,\ X[m],\ X[m+1] \,\} \\
\mathcal{X} & = & \{\, X[0],\ X[1],\ \ldots,\ X[m],\ X[m+1] \,\} \\
\bar{\mathcal{X}} & = & \{\, \bar{X}[0],\ \bar{X}[1],\ \ldots,\ \bar{X}[\bar{m}],\ \bar{X}[\bar{m}+1] \,\}
\end{array}
$$

Let $\mathrm{Mcoll}_n(M)$ denote the probability that some string is repeated in the multiset $\mathcal{X}_0$, and let $\mathrm{MMcoll}_n(M, \bar{M})$ denote the probability that some element occurs in both $\mathcal{X}$ and $\bar{\mathcal{X}}$. When m and $\bar{m}$ are numbers, let $\mathrm{Mcoll}_n(m)$ denote the maximal value of $\mathrm{Mcoll}_n(M)$ over all strings $M \in (\{0,1\}^n)^{m+2}$ and let $\mathrm{MMcoll}_n(m, \bar{m})$ denote the maximal value of $\mathrm{Mcoll}_n(M, \bar{M})$ over all $M \in (\{0,1\}^n)^{m+2}$ and $\bar{M} \in (\{0,1\}^n)^{\bar{m}+2}$ such that $M[0] \neq \bar{M}[0]$.

Think of $M[0]$ as a synonym for the nonce N, think of $M[m]$ as a generalization of $\mathrm{len}(M[m])$ (where the adversary can effectively control $M[m]$ as opposed to $\mathrm{len}(M[m])$ to influence $X[m]$), and think of $M[m+1]$ as a synonym for Checksum, which we likewise let the adversary control. One similarly understands $\bar{M}[0]$, $\bar{M}[\bar{m}]$, and $\bar{M}[\bar{m}+1]$. The needed bound is as follows.

Lemma 2 **[Bound on the M- and MM-collision probability]**

$$
\mathrm{Mcoll}_n(m) \;\leq\; \binom{m+3}{2} \cdot \frac{1}{2^n} \qquad \text{and} \qquad \mathrm{MMcoll}_n(m, \bar{m}) \;\leq\; \frac{(m+2)(\bar{m}+2)}{2^n}
$$

The CM-collision probability. The CM-collision probability is defined in Figure 4. The following lemma tells us how large it can possibly be.

Lemma 3 **[Bound on the CM-collision probability]** Assume $c, \bar{m} \leq 2^{n-2}$. Then

$$
\mathrm{CMcoll}_n(c, \bar{m}) \;\leq\; \frac{2c + 3\bar{m} + 9}{2^n}
$$

Concluding the authenticity theorem. To prove Theorem 1, combine Lemmas 1, 2, and 3. Let $\Pi = \mathrm{OCB}[\mathrm{Perm}(n), \tau]$. Given the aggregate block length σ and the bound c on the length of

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$bad \leftarrow \texttt{false};\quad \textbf{for all } x \in \{0,1\}^n \textbf{ do } \pi(x) \leftarrow \texttt{undefined}$
$L \xleftarrow{R} \{0,1\}^n;\quad \pi(0^n) \leftarrow L$

$\bar{X}[0] \leftarrow \bar{N} \oplus L;\quad \bar{Y}[0] \leftarrow \bar{R} \xleftarrow{R} \{0,1\}^n$
$\textbf{for } i \leftarrow 1 \textbf{ to } \bar{m} \textbf{ do } \bar{Z}(i) \leftarrow \gamma_i \cdot L \oplus \bar{R}$
$\textbf{for } i \leftarrow 1 \textbf{ to } \bar{m}-1 \textbf{ do } \{\ \bar{X}[i] \leftarrow \bar{M}[i] \oplus \bar{Z}[i];\quad \bar{Y}[i] \leftarrow \bar{C}[i] \oplus \bar{Z}[i]\ \}$
$\bar{X}[\bar{m}] \leftarrow \mathrm{len}(\bar{M}[\bar{m}]) \oplus huge \cdot L \oplus \bar{Z}[\bar{m}]\ ;\quad \bar{Y}(\bar{m}) \leftarrow \bar{C}[\bar{m}]\,0^* \oplus \bar{M}[\bar{m}]\,0^*$
$\mathrm{Checksum}' \leftarrow \bar{M}[1] \oplus \cdots \oplus \bar{M}[\bar{m}-1] \oplus \bar{C}[\bar{m}]\,0^* \oplus \bar{Y}[\bar{m}]$
$\bar{X}[\bar{m}+1] \leftarrow \mathrm{Checksum}' \oplus \bar{Z}[\bar{m}]$
$\textbf{for } i \leftarrow 0 \textbf{ to } \bar{m}+1 \textbf{ do } \pi(\bar{X}[i]) \leftarrow \bar{Y}[i]$

$X[0] \leftarrow N \oplus L$
$\textbf{if } N \neq \bar{N} \textbf{ and } X[0] \in \mathrm{Domain}(\pi) \textbf{ then } bad \leftarrow \texttt{true}$
$\textbf{if } N = \bar{N} \textbf{ then } R \leftarrow \bar{R} \textbf{ else } R \xleftarrow{R} \{0,1\}^n$
$\pi(X[0]) \leftarrow R$
$\textbf{for } i \leftarrow 1 \textbf{ to } c \textbf{ do } Z[i] \leftarrow \gamma_i \cdot L \oplus R$
$\textbf{for } i \leftarrow 1 \textbf{ to } c-1 \textbf{ do } \{$
$\qquad Y[i] \leftarrow C[i] \oplus Z[i]$
$\qquad \textbf{if } Y[i] \in \mathrm{Range}(\pi) \textbf{ then } X[i] \leftarrow \pi^{-1}(Y[i]) \textbf{ else } X[i] \xleftarrow{R} \{0,1\}^n$
$\qquad \pi(X[i]) \leftarrow Y[i];\quad M[i] \leftarrow X[i] \oplus Z[i]\ \}$
$X[c] \leftarrow \mathrm{len}(C[c]) \oplus huge \cdot L \oplus Z[c]$
$\textbf{if } X[c] \in \mathrm{Domain}(\pi) \textbf{ then } Y[c] \leftarrow \pi(X[c]) \textbf{ else } Y[c] \xleftarrow{R} \{0,1\}^n$
$\pi(X[c]) \leftarrow Y[c]$
$\mathrm{Checksum} \leftarrow M[1] \oplus \cdots \oplus M[c-1] \oplus C[c]\,0^* \oplus Y[c]$
$X[c+1] \leftarrow \mathrm{Checksum} \oplus Z[c]$
$\textbf{if } X[c+1] \in \mathrm{Domain}(\pi) \textbf{ then } bad \leftarrow \texttt{true}$

Figure 4: **Defining the CM-collision probability.** The function $\mathrm{CMcoll}_n(\bar{N}, \bar{M}, \bar{C},\ N, C)$ is defined as the probability that *bad* gets set to `true` when executing this game. The value $\mathrm{CMcoll}_n(c, \bar{m})$ is the maximal value of $\mathrm{CMcoll}_n(\bar{N}, \bar{M}, \bar{C},\ N, C)$ over all $\bar{m}$-block $\bar{M}$ and $\bar{C}$, and all c-block C such that $N \neq \bar{N}$ or $C \neq \bar{C}$.

the forgery attempt, one must bound the maximum possible value of

$$\begin{aligned}
\mathbf{Adv}^{\mathrm{auth}}_{\Pi}(A) &\leq \max_{\substack{m_1,\ldots,m_q \\ \sum m_i = \sigma \\ m_i \geq 1}} \left\{ \sum_{r\in[1..q]} \mathrm{Mcoll}_n(m_r) + \sum_{1\leq r<s\leq q} \mathrm{MMcoll}_n(m_r, m_s) + \sum_{r\in[1..q]} \mathrm{CMcoll}_n(c, m_r) \right\} + \\
&\qquad \frac{\bar{\sigma}^2}{2^{n+1}} + \frac{1}{2^\tau} \\
&\leq \max_{\substack{m_1,\ldots,m_q \\ \sum m_i = \sigma \\ m_i \geq 0}} \left\{ \sum_{r\in[1..q]} \frac{(m_r+3)^2}{2^{n+1}} + \sum_{1\leq r<s\leq q} \frac{(m_r+2)(m_s+2)}{2^n} + \sum_{r\in[1..q]} \left(\frac{2c+3m_r+9}{2^n} \right) \right\} + \\
&\qquad \frac{(\sigma+2q+5c+11)^2}{2^{n+1}} + \frac{1}{2^\tau}
\end{aligned}$$

One can bound the first sum by letting $m_1 = \sigma$ and letting the remaining $m_i = 0$; one can bound the second sum by letting each $m_i = \sigma/q$; and one can bound the third sum by letting $m_1 = \sigma$ and letting the remaining $m_i = 0$. These choices can be justified by the technique of Lagrange multipliers. This gives

$$\mathbf{Adv}^{\mathrm{auth}}_{\Pi}(A) \leq \frac{0.5(\sigma+3)^2 + 4.5q}{2^n} + \frac{0.5q^2(\sigma/q+2)^2}{2^n} + \frac{2c+3\sigma+9+q(2c+9)}{2^n} +$$

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$$\frac{0.5(\sigma+2q+5c+11)^2}{2^n}+\frac{1}{2^\tau}$$

$$\begin{aligned}
&\le \frac{0.5(\sigma+3)^2+4.5q+0.5(\sigma+2q)^2+2c+3\sigma+9+2cq+9q+0.5(\sigma+2q+5c+11)^2}{2^n}+\frac{1}{2^\tau}\\
&\le \frac{0.5(\sigma+3)^2+0.5(\sigma+2q)^2+0.5(\sigma+2q+5c+11)^2+(3\sigma+2cq+2c+13.5q+9)}{2^n}+\frac{1}{2^\tau}\\
&\le \frac{1.5\,(\sigma+2q+5c+11)^2}{2^n}+\frac{1}{2^\tau}\\
&\le \frac{1.5\,\bar{\sigma}^2}{2^n}+\frac{1}{2^\tau}
\end{aligned}$$

The fourth inequality can be justified by checking that $0.5(\sigma+3+(2q+5c+8))^2-0.5(\sigma+3)^2)$ already exceeds $3\sigma+2cq+2c+13.5q+9$. This completes the proof.

Privacy. Privacy is obtained rather easily en route to proving authenticity. The is because of the following result, which closely follows the first half of the proof of the structure lemma.

Lemma 4 **[Privacy lemma]** *Fix OCB parameters n and τ, and let $\Pi = \mathrm{OCB}[\mathrm{Perm}(n),\tau]$. Let A be an adversary that asks q queries, these having aggregate block length of σ blocks. Let $\mathrm{Mcoll}_n(\cdot)$ and $\mathrm{MMcoll}_n(\cdot,\cdot)$ be the M- and MM-collision probabilities. Then*

$$\mathbf{Adv}^{\mathrm{priv}}_{\Pi}(A) \;\le\; \frac{(\sigma+2q+1)^2}{2^{n+1}}+\max_{\substack{m_1,\ldots,m_q\\ \sum m_i=\sigma\\ m_i\ge 1}}\left\{\sum_{r\in[1..q]}\mathrm{Mcoll}_n(m_r)+\sum_{1\le r<s\le q}\mathrm{MMcoll}_n(m_r,m_s)\right\}$$

Combining Lemmas 2 and 4 gives Theorem 2. Namely,

$$\mathbf{Adv}^{\mathrm{priv}}_{\Pi}(A) \;\le\; \frac{(\sigma+2q+1)^2}{2^{n+1}}+\max_{\substack{m_1,\ldots,m_q\\ \sum m_i=\sigma\\ m_i\ge 0}}\left\{\sum_{r\in[1..q]}\frac{(m_r+3)^2}{2\cdot 2^n}\right\}+\max_{\substack{m_1,\ldots,m_q\\ \sum m_i=\sigma\\ m_i\ge 0}}\left\{\sum_{1\le r<s\le q}\frac{(m_r+2)(m_s+2)}{2^n}\right\}$$

and we bound the two sums exactly as before, giving

$$\begin{aligned}
\mathbf{Adv}^{\mathrm{priv}}_{\Pi}(A) &\le \frac{0.5(\sigma+2q+1)^2}{2^n}+\frac{0.5(\sigma+3)^2+4.5q}{2^n}+\frac{0.5q^2(\sigma/q+2)^2}{2^n}\\
&\le \frac{0.5(\sigma+2q+1)^2+0.5(\sigma+3)^2+4.5q+0.5(\sigma+2q)^2+4.5q}{2^n}\\
&\le \frac{1.5\,(\sigma+2q+3)^2}{2^n}\\
&\le \frac{1.5\,\bar{\sigma}^2}{2^n}
\end{aligned}$$

The third inequality can be justified by noting that $0.5(\sigma+3+2q)^2-0.5(\sigma+3)^2$ exceeds $4.5q$.

B.2 Proof of the Structure Lemma (Lemma 1)

Let A be a (computationally unbounded) adversary that attempts to violate the authenticity of $\Pi = \mathrm{OCB}[\mathrm{Perm}(n),\tau]$. Without loss of generality, A is deterministic. The adversary is given

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Initialization:

$bad \leftarrow \texttt{false}$; **for all** $x \in \{0,1\}^n$ **do** $\pi(x) \leftarrow \texttt{undefined}$
$L \xleftarrow{R} \{0,1\}^n$; $\pi(0^n) \leftarrow L$

When A **asks query** (N, M): *//q such queries will be asked*

Partition M into blocks $M[1] \cdots M[m]$
$X[0] \leftarrow N \oplus L$; $Y[0] \xleftarrow{R} \{0,1\}^n$
if $X[0] \in \mathrm{Domain}(\pi)$ **then** $\{$ $bad \leftarrow \texttt{true}$; $Y[0] \leftarrow \pi(X[0])$ $\}$ **else**
if $Y[0] \in \mathrm{Range}(\pi)$ **then** $\{$ $bad \leftarrow \texttt{true}$; $Y[0] \xleftarrow{R} \overline{\mathrm{Range}}(\pi)$ $\}$
$\pi(X[0]) \leftarrow Y[0]$

for $i \leftarrow 1$ **to** m **do** $Z[i] \leftarrow \gamma_i \cdot L \oplus Y[0]$
for $i \leftarrow 1$ **to** $m-1$ **do** $\{$
  $X[i] \leftarrow M[i] \oplus Z[i]$; $Y[i] \xleftarrow{R} \{0,1\}^n$
  if $X[i] \in \mathrm{Domain}(\pi)$ **then** $\{$ $bad \leftarrow \texttt{true}$; $Y[i] \leftarrow \pi(X[i])$ $\}$ **else**
  if $Y[i] \in \mathrm{Range}(\pi)$ **then** $\{$ $bad \leftarrow \texttt{true}$; $Y[i] \xleftarrow{R} \overline{\mathrm{Range}}(\pi)$ $\}$
  $\pi(X[i]) \leftarrow Y[i]$; $C[i] \leftarrow Y[i] \oplus Z[i]$ $\}$

$X[m] \leftarrow \mathrm{len}(M[m]) \oplus huge \cdot L \oplus Z[m]$; $Y[m] \xleftarrow{R} \{0,1\}^n$
if $X[m] \in \mathrm{Domain}(\pi)$ **then** $\{$ $bad \leftarrow \texttt{true}$; $Y[m] \leftarrow \pi(X[m])$ $\}$ **else**
if $Y[m] \in \mathrm{Range}(\pi)$ **then** $\{$ $bad \leftarrow \texttt{true}$; $Y[m] \xleftarrow{R} \overline{\mathrm{Range}}(\pi)$ $\}$
$\pi(X[m]) \leftarrow Y[m]$; $C[m] \leftarrow M[m] \oplus Y[m]$

$\mathrm{Checksum} \leftarrow M[1] \oplus \cdots \oplus M[m-1] \oplus C[m]\, 0^* \oplus Y[m]$
$X[m+1] \leftarrow \mathrm{Checksum} \oplus Z[m]$; $Y[m+1] \xleftarrow{R} \{0,1\}^n$
if $X[m+1] \in \mathrm{Domain}(\pi)$ **then** $\{$ $bad \leftarrow \texttt{true}$; $Y[m+1] \leftarrow \pi(X[m+1])$ $\}$ **else**
if $Y[m+1] \in \mathrm{Range}(\pi)$ **then** $\{$ $bad \leftarrow \texttt{true}$; $Y[m+1] \xleftarrow{R} \overline{\mathrm{Range}}(\pi)$ $\}$
$\pi(X[m+1]) \leftarrow Y[m+1]$; $T \leftarrow Y[m+1]$ [first τ bits]
return $\mathcal{C} \leftarrow C[1] \cdots C[m]\, T$

Figure 5: **Game A, part 1.** This game provides adversary A a perfect simulation of $\mathrm{OCB}[\mathrm{Perm}(n), \tau]$.

an oracle for $\mathrm{OCB.Enc}_\pi(\cdot,\cdot)$. We must bound the probability that A, after adaptively using this oracle q times, on messages with aggregate length σ blocks, produces a properly forged ciphertext having at most c blocks. This forgery probability is denoted $\mathbf{Adv}^{\mathrm{auth}}_{\Pi}(A)$.

Game A. One can conceive of A interacting with $\mathrm{OCB.Enc}_\pi(\cdot,\cdot)$ and then producing a forgery attempt as A playing a certain game, game A, as defined in Figures 5 and 6. Rather than choose $\pi \xleftarrow{R} \mathrm{Perm}(n)$ all at once, this game defines the values of $\pi(x)$ point-by-point, as needed. We use the notation $\mathrm{Domain}(\pi)$ for the set of values $x \in \{0,1\}^n$ such that $\pi(x) \neq \texttt{undefined}$. By $\overline{\mathrm{Domain}}(\pi)$ we mean $\{0,1\}^n \setminus \mathrm{Domain}(\pi)$. Similarly, $\mathrm{Range}(\pi)$ is the set of $y \in \{0,1\}^n$ such that there exists an $x \in \{0,1\}^n$ for which $\pi(x) = y$, and $\overline{\mathrm{Range}}(\pi) = \{0,1\}^n \setminus \mathrm{Range}(\pi)$.

An inspection of game A makes clear that it supplies to A a perfect simulation of $\mathrm{OCB.Enc}_\pi(\cdot,\cdot)$. Game A simulates OCB in a somewhat unusual way, not only defining π point-by-point, but, when a value $\pi(x)$ is needed, for some new x, we get this value, in most cases, not by choosing $y \xleftarrow{R} \overline{\mathrm{Range}}(\pi)$, as would seem natural, but by choosing $y \xleftarrow{R} \{0,1\}^n$, setting $\pi(x)$ to y if y is not already in the range of π, and "changing our minds," setting $\pi(x) \xleftarrow{R} \overline{\mathrm{Range}}(\pi)$, otherwise. In the latter case, a flag *bad* is set to `true`. The flag *bad* is also set to `true` when the adversary successfully forges. Consequently, upperbounding the probability that *bad* gets set to `true` in game A serves

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When A **makes forgery attempt** $(N,\ \mathcal{C})$:
50 Partition $\mathcal{C}$ into $C[1]\cdots C[c]\ T$
51 $X[0] \leftarrow N \oplus L$; **if** $X[0] \in \mathrm{Domain}(\pi)$ **then** $Y[0] \leftarrow \pi(X[0])$ **else** $Y[0] \xleftarrow{R} \overline{\mathrm{Range}}(\pi)$
52 $\pi(X[0]) \leftarrow Y[0]$
53 **for** $i \leftarrow 1$ **to** c **do** $Z[i] \leftarrow \gamma_i \cdot L \oplus Y[0]$
54 **for** $i \leftarrow 1$ **to** $c-1$ **do** {
55 $Y[i] \leftarrow C[i] \oplus Z[i]$
56 **if** $Y[i] \in \mathrm{Range}(\pi)$ **then** $X[i] \leftarrow \pi^{-1}(Y[i])$ **else** $X[i] \xleftarrow{R} \overline{\mathrm{Domain}}(\pi)$
57 $\pi(X[i]) \leftarrow Y[i]$; $M[i] \leftarrow X[i] \oplus Z[i]$ }

58 $X[c] \leftarrow \mathrm{len}(C[c]) \oplus \mathit{huge} \cdot L \oplus Z[c]$
59 **if** $X[c] \in \mathrm{Domain}(\pi)$ **then** $Y[c] \leftarrow \pi(X[c])$ **else** $Y[c] \xleftarrow{R} \overline{\mathrm{Range}}(\pi)$
60 $\pi(X[c]) \leftarrow Y[c]$
61 $\mathrm{Checksum} \leftarrow M[1] \oplus \cdots \oplus M[c-1] \oplus C[c]\,0^* \oplus Y[c]$
62 $X[c+1] \leftarrow \mathrm{Checksum} \oplus Z[c]$
63 **if** $X[c+1] \in \mathrm{Domain}(\pi)$ **then** $Y[c+1] \leftarrow \pi(X[c])$ **else** $Y[c+1] \xleftarrow{R} \overline{\mathrm{Range}}(\pi)$
64 $T' \leftarrow Y[c+1]$ [first τ bits]
65 **if** $T = T'$ **then** $\mathit{bad} \leftarrow$ `true`

Figure 6: **Games A, A′ B, B′, and C, part 2.**

to upperbound the adversary's forging probability.

Game A′. We begin by making a couple of quite trivial changes to game A. First, instead of setting $C[m] = M[m] \oplus Y[m]$ (in line 24 of game A), we set $C[m] = M[m]\,0^* \oplus Y[m]$, instead. That is, we imagine returning the "full" final-ciphertext-block instead of the truncated final-ciphertext-block. Clearly the extra bits given to the adverary can not make worse an optimal adversary's chance of successful forgery. Second, instead of returning (in line 30 of game A) a tag T which is the first τ bits of $Y[m+1]$, we return the full tag, $Y[m+1]$. Once again, the extra bits provided to the adverary can only improve an optimal adversary's chance of success. Let game A′ denote this new, "easier" game. We will bound the probability that *bad* gets set to `true` in game A′.

Game B. Next we eliminate from game A′ the statement which immediately follows *bad* being set to `true` in each of lines 12, 13, 18, 19, 22, 23, 27, 28. The **else** statements are also eliminated. This new game, game B, is shown in Figure 7. This new game is different from game A′, and an adversary A having queries answered according to game B will not be seeing the same view as one whose queries are answered according to A′. Still, game B has been constructed so that it behaves identically to game A′ until the flag *bad* is set to `true`. Only at that point do the two games diverge. As a consequence, regardless of the behavior of A, the probaiblity that *bad* will get set to `true` when A plays game B is identical to the probability that *bad* gets set to `true` when A plays game A′. Now we are interested in upperbounding the probability of forgery in game A, which we do by upperbounding the probability that *bad* gets set to `true` in game A′, which is just the probability that *bad* gets set to `true` in game B.

Note that we are not claiming that the probability of the adversary forging in game B (meaning that *bad* gets set to `true` at line 65 of game B) is the same as the probability of the adversary forging in A′ (meaning that *bad* gets set to `true` in the last line of that game). Claims of this sort are tempting to make, but they are untrue.

Bounding Y-collisions in Game B. We next bound the probability that *bad* will be set to `true`

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Initialization:
01 $\quad bad \leftarrow$ `false`; **for all** $x \in \{0,1\}^n$ **do** $\pi(x) \leftarrow$ `undefined`
02 $\quad L \xleftarrow{R} \{0,1\}^n$; $\pi(0^n) \leftarrow L$

When A **asks query** (N, M): $\qquad$ *//q such queries will be asked*
10 $\quad$ Partition M into blocks $M[1] \cdots M[m]$
11 $\quad X[0] \leftarrow N \oplus L$; $Y[0] \xleftarrow{R} \{0,1\}^n$
12 $\quad$ **if** $X[0] \in \text{Domain}(\pi)$ **then** $bad \leftarrow$ `true`
13 $\quad$ **if** $Y[0] \in \text{Range}(\pi)$ **then** $bad \leftarrow$ `true`
14 $\quad \pi(X[0]) \leftarrow Y[0]$
15 $\quad$ **for** $i \leftarrow 1$ **to** m **do** $Z[i] \leftarrow \gamma_i \cdot L \oplus Y[0]$
16 $\quad$ **for** $i \leftarrow 1$ **to** $m-1$ **do** $\{$
17 $\qquad X[i] \leftarrow M[i] \oplus Z[i]$; $Y[i] \xleftarrow{R} \{0,1\}^n$
18 $\qquad$ **if** $X[i] \in \text{Domain}(\pi)$ **then** $bad \leftarrow$ `true`
19 $\qquad$ **if** $Y[i] \in \text{Range}(\pi)$ **then** $bad \leftarrow$ `true`
20 $\qquad \pi(X[i]) \leftarrow Y[i]$; $C[i] \leftarrow Y[i] \oplus Z[i]$ $\}$
21 $\quad X[m] \leftarrow \text{len}(M[m]) \oplus \mathit{huge} \cdot L \oplus Z[m]$; $Y[m] \xleftarrow{R} \{0,1\}^n$
22 $\quad$ **if** $X[m] \in \text{Domain}(\pi)$ **then** $bad \leftarrow$ `true`
23 $\quad$ **if** $Y[m] \in \text{Range}(\pi)$ **then** $bad \leftarrow$ `true`
24 $\quad \pi(X[m]) \leftarrow Y[m]$; $C[m] \leftarrow M[m]\,0^* \oplus Y[m]$
25 $\quad \text{Checksum} \leftarrow M[1] \oplus \cdots \oplus M[m-1] \oplus C[m]\,0^* \oplus Y[m]$
26 $\quad X[m+1] \leftarrow \text{Checksum} \oplus Z[m]$; $Y[m+1] \xleftarrow{R} \{0,1\}^n$
27 $\quad$ **if** $X[m+1] \in \text{Domain}(\pi)$ **then** $bad \leftarrow$ `true`
28 $\quad$ **if** $Y[m+1] \in \text{Range}(\pi)$ **then** $bad \leftarrow$ `true`
29 $\quad \pi(X[m+1]) \leftarrow Y[m+1]$
30 $\quad$ **return** $\mathcal{C} \leftarrow C[1] \cdots C[m]\; Y[m+1]$

Figure 7: **Game B, part 1.**

in any of lines 13, 19, 23, or 28 of game B. In each of these lines, a random n-bit string was just chosen and then it is tested for membership in the growing set Range(π). In the course of game B the size Range(π) starts off at 0 and then grows one element at a time until it reaches a final size of $\sigma + 2q + 1$ elements. Therefore the probability that, in growing Range(π), there is a repetition as we add in random points is at most $(1 + 2 + \cdots + \sigma + 2q)/2^n \le (\sigma + 2q + 1)^2/2^{n+1}$. We note this for future reference:

$$\Pr[A \text{ causes } \mathit{bad} \text{ to be set in any of lines 13, 19, 23 or 28 of game B}] \;\le\; \frac{(\sigma + 2q + 1)^2}{2^{n+1}} \qquad (1)$$

Having bounded the probability that *bad* will be set in the four indicated lines, we may imagine eliminating these four lines, forming a new game, game B′. The probability that *bad* is set in game B is at most the computed bound more than than the probability that *bad* is set in game B′. Thus we may continue the analysis using game B′ as long as we compensate the final bound by adding in the term given by Equation (1).

Game C. In game B′, consider the distribution on strings returned to the adversary in response to a query $(N,\, M)$, where $m = \|M\|_n$. The adversary learns $\mathcal{C} = C[1] \cdots C[m-1]C[m]\; Y[m+1]$. Since each block of this string is a uniform random value xor'ed with some other, independent value, we have that $\mathcal{C}$ is uniformly distributed and independent of the query M, apart from its length. As a consequence, when a query of N, M is made, where M has m blocks, we can return a random answer $\mathcal{C}$ (of $nm + n$ bits) and do no more at that time. Later, when the adversary is done making

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When A asks its r-th query, $(N_r,\ M_r)$: *//r will range from 1 to q*
10 Partition M_r into blocks $M_r[1] \cdots M_r[m_r]$
11 $C_r[1], \ldots, C_r[m_r],\ Y_r[m+1] \stackrel{R}{\leftarrow} \{0,1\}^n$
12 **return** $\mathcal{C}_r \leftarrow C_r[1] \cdots C_r[m_r]\ Y_r[m_r+1]$

When A is done making oracle queries:
20 $bad \leftarrow$ `false`; **for all** $x \in \{0,1\}^n$ **do** $\pi(x) \leftarrow$ `undefined`
21 $L \stackrel{R}{\leftarrow} \{0,1\}^n$; $\pi(0^n) \leftarrow L$

30 **for** $r \leftarrow 1$ **to** q **do** {
31 $X_r[0] \leftarrow N_r \oplus L$; $Y_r[0] \stackrel{R}{\leftarrow} \{0,1\}^n$
32 **for** $i \leftarrow 1$ **to** m_r **do** $Z_r[i] \leftarrow \gamma_i \cdot L \oplus Y_r[0]$
33 **for** $i \leftarrow 1$ **to** $m_r - 1$ **do** { $X_r[i] \leftarrow M_r[i] \oplus Z_r[i]$; $Y_r[i] \leftarrow C_r[i] \oplus Z_r[i]$ }
34 $X_r[m_r] \leftarrow \mathrm{len}(M[m_r]) \oplus huge \cdot L \oplus Z_r[m_r]$; $Y_r[m_r] \leftarrow C_r[m_r] \oplus M_r[m_r]\,0^*$
35 $\mathrm{Checksum}_r \leftarrow M_r[1] \oplus \cdots \oplus M_r[m_r-1] \oplus C_r[m_r]\,0^* \oplus Y_r[m_r]$
36 $X_r[m_r+1] \leftarrow \mathrm{Checksum}_r \oplus Z_r[m_r]$ }

37 $\mathcal{X} \leftarrow (X_1[0], X_1[1], \ldots, X_1[m_1+1],\ \ldots,\ X_q[0], X_q[1], \ldots, X_q[m_q+1])$
38 $\mathcal{Y} \leftarrow (Y_1[0], Y_1[1], \ldots, Y_1[m_1+1],\ \ldots,\ Y_q[0], Y_q[1], \ldots, Y_q[m_q+1])$
39 **if** some string is repeated in $\mathcal{X} \cup \{0^n\}$ **then** $bad \leftarrow$ `true`
40 **for** $i \leftarrow 1$ **to** $|\mathcal{X}|$ **do** $\pi(\mathcal{X}[i]) \leftarrow \mathcal{Y}[i]$

Figure 8: **Game C, part 1.** This game provides adversary A with the same view as game B, and sets *bad* with the same probability. But it defers some random choices.

its q queries, we can set the remaining random values, make the associated assignments to π, and set the flag *bad*, as appropriate. This is what has been done in Game C of Figure 8. From the adversary's point of view, game B$'$ and game C are identical. Furthermore, the probability that *bad* gets set to `true` is identical in the two games.

Game D. We have reduced the problem of upperbounding the forging probability to the problem of upperbounding the probability that *bad* gets set to `true` in game C. This probability is over the coins used in line 11 of game C (which defines the $\mathcal{C}_r$-values) and over the additional coins used subsequently in the program. We must show that, over this sequence of coins (remember that the adversary is deterministic) the flag *bad* is rarely set.

We will show something stronger: that even if one fixes all of the coins used in line 11 (the $\mathcal{C}_r$-values) and takes the probability over just the remaining coins, still the probability that *bad* gets set to `true` is small. The virtue of this change is that it effectively eliminates the q interactive queries from the game. Namely, since the adversary A is deterministic and each response $\mathcal{C}_r$ has been fixed, the adversary can be imagined to "know" all of the queries $N_1, M_1,\ \ldots,\ N_q, M_q$ that it would ask and all of the answers $\mathcal{C}_1, \ldots, \mathcal{C}_q$ that it would receive. All the adversary has left to do is to output the forgery attempt $(N, C\ T)$. This value too is now pre-determined, as our adversary is deterministic. So the adversary is effectively gone, and we are left to claim that for any $N_1, M_1, \ldots, N_q, M_q,\ \mathcal{C}_1, \ldots, \mathcal{C}_q,\ N, C, T$, the flag *bad* will rarely be set if we run game C starting at line 20. The new game is called game D. It depends on $N_1, M_1, \ldots, N_q, M_q,\ \mathcal{C}_1, \ldots, \mathcal{C}_q,\ N, C, T$, which are now just constants. The constants are not quite arbitrary: the N_r-values are still required to be distinct. The lengths of $M_1, \ldots, M_q$ are $m_1, \ldots, m_q$ blocks. The length of C is c blocks.

The Mcoll_n and MMcoll_n terms. At this point we make the observation that *bad* will be set to

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20 $\mathit{bad} \leftarrow \texttt{false}$; **for all** $x \in \{0,1\}^n$ **do** $\pi(x) \leftarrow \texttt{undefined}$
21 $L \xleftarrow{R} \{0,1\}^n$; $\pi(0^n) \leftarrow L$

30 **for** $r \leftarrow 1$ **to** q **do** {
31 $\quad X_r[0] \leftarrow N_r \oplus L$; $Y_r[0] \xleftarrow{R} \{0,1\}^n$
32 $\quad$ **for** $i \leftarrow 1$ **to** m_r **do** $Z_r[i] \leftarrow \gamma_i \cdot L \oplus Y_r[0]$
33 $\quad$ **for** $i \leftarrow 1$ **to** $m_r - 1$ **do** { $X_r[i] \leftarrow M_r[i] \oplus Z_r[i]$; $Y_r[i] \leftarrow C_r[i] \oplus Z_r[i]$ }
34 $\quad X_r[m_r] \leftarrow \mathrm{len}(M[m_r]) \oplus \mathit{huge} \cdot L \oplus Z_r[m_r]$; $Y_r[m_r] \leftarrow C_r[m_r] \oplus M_r[m_r]\, 0^*$
35 $\quad \mathrm{Checksum}_r \leftarrow M_r[1] \oplus \cdots \oplus M_r[m_r - 1] \oplus C_r[m_r]\, 0^* \oplus Y_r[m_r]$
36 $\quad X_r[m_r + 1] \leftarrow \mathrm{Checksum}_r \oplus Z_r[m_r]$ }
37 $\mathcal{X} \leftarrow (X_1[0], X_1[1], \ldots, X_1[m_1 + 1], \ \ldots, \ X_q[0], X_q[1], \ldots, X_q[m_q + 1])$
38 $\mathcal{Y} \leftarrow (Y_1[0], Y_1[1], \ldots, Y_1[m_1 + 1], \ \ldots, \ Y_q[0], Y_q[1], \ldots, Y_q[m_q + 1])$
39 **for** $i \leftarrow 1$ **to** $|\mathcal{X}|$ **do** $\pi(\mathcal{X}[i]) \leftarrow \mathcal{Y}[i]$
40 **if** some string is repeated in $\mathcal{X} \cup \{0^n\}$ **then** $\mathit{bad} \leftarrow \texttt{true}$

50 $X[0] \leftarrow N \oplus L$; **if** $X[0] \in \mathrm{Domain}(\pi)$ **then** $Y[0] \leftarrow \pi(X[0])$ **else** $Y[0] \xleftarrow{R} \overline{\mathrm{Range}}(\pi)$
51 $\pi(X[0]) \leftarrow Y[0]$
52 **for** $i \leftarrow 1$ **to** c **do** $Z[i] \leftarrow \gamma_i \cdot L \oplus Y[0]$
53 **for** $i \leftarrow 1$ **to** $c - 1$ **do** {
54 $\quad Y[i] \leftarrow C[i] \oplus Z[i]$
55 $\quad$ **if** $Y[i] \in \mathrm{Range}(\pi)$ **then** $X[i] \leftarrow \pi^{-1}(Y[i])$ **else** $X[i] \xleftarrow{R} \overline{\mathrm{Domain}}(\pi)$
56 $\quad \pi(X[i]) \leftarrow Y[i]$; $M[i] \leftarrow X[i] \oplus Z[i]$ }
57 $X[c] \leftarrow \mathrm{len}(C[c]) \oplus \mathit{huge} \cdot L \oplus Z[c]$
58 **if** $X[c] \in \mathrm{Domain}(\pi)$ **then** $Y[c] \leftarrow \pi(X[c])$ **else** $Y[c] \xleftarrow{R} \overline{\mathrm{Range}}(\pi)$
59 $\pi(X[c]) \leftarrow Y[c]$
60 $\mathrm{Checksum} \leftarrow M[1] \oplus \cdots \oplus M[c-1] \oplus C[c]\, 0^* \oplus Y[c]$
61 $X[c+1] \leftarrow \mathrm{Checksum} \oplus Z[c]$
62 **if** $X[c+1] \in \mathrm{Domain}(\pi)$ **then** $Y[c+1] \leftarrow \pi(X[c])$ **else** $Y[c+1] \xleftarrow{R} \overline{\mathrm{Range}}(\pi)$
63 $T' \leftarrow Y[c+1]$ [first τ bits]
64 **if** $T = T'$ **then** $\mathit{bad} \leftarrow \texttt{true}$

Figure 9: **Game D.** This game depends on $N_1, \ldots, N_q$, $M_1, \ldots, M_q$, $C_1, \ldots, C_q$, $Y_1[m_1+1], \ldots, Y_q[m_q+1]$, N, $C = C[1] \cdots C[c]$ and T.

`true` in line 40 of game D if and only if either

- There is some $r \in [1..q]$ such that there is a repetition in the multiset

$$\{0^n, \ X_r[0], \ X_r[1], \ \ldots, X_r[m_r]\}$$

- There is some pair $r, s \in [1..q]$, where $r < s$, such that

$$\{X_r[0], \ \ldots \ X_r[m_r + 1]\} \quad \text{has some a point in common with} \quad \{X_s[0], \ \ldots \ X_s[m_s + 1]\}$$

The probability of this event is at most

$$\sum_{r \in [1..q]} \mathrm{Mcoll}_n(m_r) + \sum_{1 \le r < s \le q} \mathrm{MMcoll}_n(m_r, m_s) \qquad (2)$$

by our definition of Mcoll_n and MMcoll_n. Therefore the probability that *bad* is set to `true` in line 40 of Game D is at most the expression above. We are left now to focus on the probability that *bad* gets set to `true` in line 64 of Game D (Figures 9 and 6).

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50 $X[0] \leftarrow N \oplus L$
51 **if** $N \neq N_r$ for any r **and** $X[0] \in \mathrm{Domain}(\pi)$ **then** $bad \leftarrow \texttt{true}$
52 **if** $N = N_r$ for some r **then** $Y[0] \leftarrow Y_r[0]$ **else** $Y[0] \stackrel{R}{\leftarrow} \{0,1\}^n$
53 $\pi(X[0]) \leftarrow Y[0]$
54 **for** $i \leftarrow 1$ **to** c **do** $Z[i] \leftarrow \gamma_i \cdot L \oplus Y[0]$
55 **for** $i \leftarrow 1$ **to** $c-1$ **do** $\{$
56     $Y[i] \leftarrow C[i] \oplus Z[i]$
57   **if** $Y[i] \in \mathrm{Range}(\pi)$ **then** $X[i] \leftarrow \pi^{-1}(Y[i])$ **else** $X[i] \stackrel{R}{\leftarrow} \{0,1\}^n$
58     $\pi(X[i]) \leftarrow Y[i]; \quad M[i] \leftarrow X[i] \oplus Z[i]\ \}$
59 $X[c] \leftarrow \mathrm{len}(C[c]) \oplus huge \cdot L \oplus Z[c]$
60 **if** $X[c] \in \mathrm{Domain}(\pi)$ **then** $Y[c] \leftarrow \pi(X[c])$ **else** $Y[c] \stackrel{R}{\leftarrow} \{0,1\}^n$
61 $\pi(X[c]) \leftarrow Y[c]$
62 $\mathrm{Checksum} \leftarrow M[1] \oplus \cdots \oplus M[c-1] \oplus C[c]\,0^* \oplus Y[c]$
63 $X[c+1] \leftarrow \mathrm{Checksum} \oplus Z[c]$
64 **if** $X[c+1] \in \mathrm{Domain}(\pi)$ **then** $bad \leftarrow \texttt{true}$

Figure 10: **Game E, part 2.** The first half of this game is lines 20–39 of Game D.

GAME E. We modify the second half of game D (lines 20–39 are unchanged). First, we simplify lines 50, 55 and 58, and 62 by choosing a random value in $\{0,1\}^n$ as opposed to a value in the co-range, co-domain, co-range, and co-range of π, respectively. By similar reasoning to that used before, this new game may decrease the probability that *bad* gets set to `true`, but by an amount that is at most

$$\frac{(c+2)(\sigma+2q+c+3)}{2^n}$$

Second, we modify the game so as to "give up" (set *bad*) if the condition of line 62 is satisfied. (Here is where pretag-collisions would begin to cause extra complications.) In doing this, we may again decrease the probability that *bad* will be set to `true`. But the decrease is at most $1/2^\tau$ since, when the **else** clause of the new line 62 is executed (that is, $Y[m+1] \stackrel{R}{\leftarrow} \{0,1\}^n$), T will equal T' with probability exactly $1/2^\tau$. Finally, we modify the game to give up (set *bad*) whenever $N \notin \{N_1, \ldots, N_q\}$ but $X[0] = N \oplus L$ is already in $\mathrm{Domain}(\pi)$ when this is checked at line 50. The new game is called game E and it is shown in Figure 10. We note for future reference:

$$\begin{aligned} &\Pr[bad \text{ gets set in game D}] \\ &\quad \leq \Pr[bad \text{ gets set in game E}] + \frac{(c+2)(\sigma+2q+c+3)^2}{2^n} + \frac{1}{2^\tau} \end{aligned} \tag{3}$$

GAME F. We now examine game E and relate it to a final game, F. If *bad* is set to `true` in game E the reason is either that $X[0] = N \oplus L$ was found to be in the domain of π even though N is a new nonce, or else $X[c+1]$ was found to be in the domain of π when this was checked. In the latter case, how did $X[c+1]$ come to be in the domain of π? At least one of the following must be true:

- $X[c+1] = 0^n$. (The value 0^n was added to the domain of π at line 21.)
- For some $r \in [1..q]$, for some $j \in [0..m_r+1]$, $X[c+1] = X_r[j]$. (These values were added to the domain of π at line 39.)
- For some $i \in [0..c]$, $X[c+1] = X[i]$. (These values were added to the domain of π at lines 53, 57, and 61).

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When *bad* is set to `true` we will assign responsibility for this event to exactly one index $r \in [1..q]$. We say that the *responsible index* is r where:

- If N is a new nonce and $X[0] \in \mathrm{Domain}(\pi)$ at line 51, then the responsible index is the least $r \in [1..q]$ such that $X_r[j] = X[0]$ for some j. Otherwise,
- If $X[c+1] = 0^n$, then the responsible index is $r = 1$. Otherwise,
- If there is an $r \in [1..q]$ such that, for some $j \in [0..m_r{+}1]$, $X[c{+}1] = X_r[j]$, then the responsible index is the least such value r. Otherwise,
- The responsible index is $r = 1$. (This last case can happen when $X[c+1] = X[i]$ for some $i \in [0..c]$.)

Partition the coins used in the running of game E into: the coins s_0 used in the initialization step (line 21); the coins $s_1, \ldots, s_q$ used for processing message $M_1, \ldots, M_q$, respectively (line 31); and the coins s used to process the forgery attempt C (lines 52, 57, and 60). Suppose we eliminate the **for** statement at line 30, and execute lines 31–36 for some specific value of r. Call this game E_r. We make the crucial observation that if *bad* is set to `true` in game E using coins $(s_0, s_1, \ldots, s_q, s)$ then *bad* will *still* be set to `true` in game E_r using coins (s_0, s_r, s) when the responsible index is r. This follows from our definition of the responsible index. The only observation that is needed is that when $X[c+1] = X[i]$ for some $i \in [0..c]$, then, considering the least such i, if $X[i]$ was selected by assigning to it an already-selected $X_s[j]$-value, then the third case in the definition of the responsible index will result in the selection of an index r that forces *bad* to `true`.

By what we have said, one can bound the probability that *bad* gets set to `true` in game E by summing the probabilities that *bad* gets set to `true` in game E_r, where $r \in [1..q]$. Game E_r is precisely the game that was used to define the CMcoll_n; in particular, the probability that *bad* is set in E_r is $\mathrm{CMcoll}_n(c, m_r)$. We conclude that the probability that *bad* is set to `true` in game E_r is at most $\mathrm{CMcoll}_n(c, m_r)$. Thus the probability that *bad* gets set to `true` in game E is at most

$$\sum_{r=1}^{q} \mathrm{CMcoll}_n(c, m_r) \tag{4}$$

Summing Equations (1), (2), (3) and (4) gives that the adversary's chance of forgery is at most

$$\sum_{r\in[1..q]} \mathrm{Mcoll}_n(m_r) + \sum_{1\le r<s\le q} \mathrm{MMcoll}_n(m_r, m_s) + \sum_{r=1}^{q} \mathrm{CMcoll}_n(c, m_r) + \frac{(\sigma+2q+1)^2 + 2(c+2)(\sigma+2q+c+3)}{2^{n+1}} + \frac{1}{2^\tau}$$

Using that $(\sigma+\Delta)^2 - \sigma^2 \ge 2\sigma\Delta$ and $(\sigma+\Delta)^2 - \sigma^2 \ge \Delta^2$, we can increase σ by a small amount in order to compensate for the lower-order terms and clean up the expression. Namely, increasing σ by $2q+1$ is enough to take care of the first addend, while increasing σ by $c+2$ plus $2(c+2)$ plus $\sqrt{2}(c{+}3)$ is enough to take care of the second addend. So increasing σ by $2q+5c+11$ will take care of both. Letting $\bar{\sigma} = 2q+5c+11$ we thus have that the adversary's chance of forgery is at most

$$\sum_{1\le r<s\le q} \mathrm{MMcoll}_n(q_r, q_s) + \sum_{r=1}^{q} \mathrm{CMcoll}_n(c, q_s) + \frac{\bar{\sigma}^2}{2} \cdot \frac{1}{2^n} + \frac{1}{2^\tau}$$

This completes the proof of the structure lemma.

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B.3 Proof of the M- and MM-Collision Bounds (Lemma 2)

We assume that $m, \bar{m} < 2^{n-2}$, since the specified probability upper bound is meaningless (it exceeds 1) otherwise. According to remarks we have made earlier, this ensures that $\gamma_1, \ldots, \gamma_{\max\{m,\bar{m}\}}, \mathit{huge}$ are distinct nonzero field elements.

We begin with the first inequality. There are $m+3$ points in the set $\mathcal{X}_0$, and we claim that for any two of them, the probability that they coincide is at most $1/2^n$. This is enough to show the first inequality, that the probability of a collision within $\mathcal{X}_0$ is at most $\binom{m+3}{2} \cdot 2^{-n}$. There are a few cases to consider. Below, remember that L and R are random, and everything else is constant. The probabilities are over L, R. In the following, we let $i, i' \in [1..m-1]$, $i \neq i'$.

- $\Pr[X[-1] = X[0]] = \Pr[0^n = M[0] \oplus L] = 1/2^n$.
- $\Pr[X[-1] = X[i]] = \Pr[0^n = M[i] \oplus \gamma_i \cdot L \oplus R] = 1/2^n$.
- $\Pr[X[-1] = X[m]] = \Pr[0^n = M[m] \oplus (\gamma_m \oplus \mathit{huge}) \cdot L \oplus R] = 1/2^n$.
- $\Pr[X[-1] = X[m+1]] = \Pr[0^n = M[m+1] \oplus \gamma_m \cdot L \oplus R] = 1/2^n$.
- $\Pr[X[0] = X[i]] = \Pr[M[0] \oplus L = M[i] \oplus \gamma_i \cdot L \oplus R] = 1/2^n$.
- $\Pr[X[0] = X[m]] = \Pr[M[0] \oplus L = M[m] \oplus (\gamma_m \oplus \mathit{huge}) \cdot L \oplus R] = 1/2^n$.
- $\Pr[X[0] = X[m+1]] = \Pr[M[0] \oplus L = M[m+1] \oplus \gamma_m \cdot L \oplus R] = 1/2^n$.
- $\Pr[X[i] = X[i']] = \Pr[M[i] \oplus \gamma_i \cdot L = M[i'] \oplus \gamma_{i'} \cdot L] = \Pr[M[i] \oplus M[i'] = (\gamma_i \oplus \gamma_{i'}) \cdot L] = 1/2^n$ because $\gamma_i \neq \gamma_{i'}$.
- $\Pr[X[i] = X[m]] = \Pr[M[i] \oplus \gamma_i \cdot L \oplus R = M[m] \oplus (\gamma_m \oplus \mathit{huge}) \cdot L \oplus R] = \Pr[M[i] \oplus \gamma_i \cdot L = M[m] \oplus (\gamma_m \oplus \mathit{huge}) \cdot L] = \Pr[M[i] \oplus M[m] = (\gamma_m \oplus \mathit{huge} \oplus \gamma_i) \cdot L] = 1/2^n$ because $\gamma_i \oplus \gamma_m \neq \mathit{huge}$. The reason that $\gamma_i \oplus \gamma_m \neq \mathit{huge}$ is that *huge* begins with a 1 in bit position 1, while neither γ_i nor γ_m do, because $i, m \leq 2^{n-2}$ and $\gamma_i < 2i$, $\gamma_m \leq 2m$.
- $\Pr[X[i] = X[m+1]] = \Pr[M[i] \oplus \gamma_i \cdot L \oplus R = M[m+1] \oplus \gamma_m \cdot L \oplus R] = \Pr[M[i] \oplus M[m+1] = (\gamma_i \oplus \gamma_m) \cdot L] = 1/2^n$.
- $\Pr[X[m] = X[m+1]] = \Pr[M[m] \oplus (\gamma_m \oplus \mathit{huge}) \cdot L \oplus R = M[m+1] \oplus \gamma_m \cdot L \oplus R] = \Pr[M[m] \oplus M[m+1] = \mathit{huge} \cdot L] = 1/2^n$.

This completes the first inequality.

For the second inequality, we wish to show that for any point in $\mathcal{X}$ and any point in $\bar{\mathcal{X}}$, the probability that they coincide is at most 2^{-n}. The result follows, since there are at most $(m+2)(\bar{m}+2)$ such pairs. Remember, below, that L, R and $\bar{R}$ are random, and everything else is constant. We let $i \in [1..m-1]$ and $j \in [1..\bar{m}-1]$. As before, $\gamma_1, \ldots, \gamma_m, \mathit{huge}$ are distinct nonzero points.

- $\Pr[X[0] = \bar{X}[0]] = \Pr[M[0] \oplus L = \bar{M}[0] \oplus L] = 0$, since $M[0] \neq \bar{M}[0]$ by assumption.
- $\Pr[X[0] = \bar{X}[j]] = \Pr[M[0] \oplus L = \bar{M}[j] \oplus \gamma_j \cdot L \oplus \bar{R}] = 1/2^n$ due to the influence of $\bar{R}$.
- $\Pr[X[0] = \bar{X}[\bar{m}] = \Pr[M[0] \oplus L = \bar{M}[\bar{m}] \oplus (\gamma_{\bar{m}} \oplus \mathit{huge}) \cdot L \oplus \bar{R}] = 1/2^n$ due to the influence of $\bar{R}$.
- $\Pr[X[0] = \bar{X}[\bar{m}+1] = \Pr[M[0] \oplus L = \bar{M}[\bar{m}+1] \oplus \gamma_{\bar{m}} \cdot L \oplus \bar{R}] = 1/2^n$ due to the influence of $\bar{R}$.
- $\Pr[X[i] = \bar{X}[j]] = \Pr[M[i] \oplus \gamma_i \cdot L \oplus R = \bar{M}[j] \oplus \gamma_j \cdot L \oplus \bar{R}] = 1/2^n$ due to the influence of $\bar{R}$.
- $\Pr[X[i] = \bar{X}[\bar{m}]] = \Pr[M[i] \oplus \gamma_i \cdot L \oplus R = \bar{M}[\bar{m}] \oplus (\gamma_{\bar{m}} \oplus \mathit{huge}) \cdot L \oplus \bar{R}] = 1/2^n$ due to the influence of $\bar{R}$.
- $\Pr[X[i] = \bar{X}[\bar{m}+1] = \Pr[M[i] \oplus \gamma_i \cdot L \oplus R = \bar{M}[\bar{m}+1] \oplus \gamma_{\bar{m}} \cdot L \oplus \bar{R}] = 1/2^n$ due to the influence of $\bar{R}$.
- $\Pr[X[m] = \bar{X}[\bar{m}] = 1/2^n$, as before, due to the influence of $\bar{R}$.
- $\Pr[X[m] = \bar{X}[\bar{m}+1] = 1/2^n$ for the same reason.

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- $\Pr[X[m+1] = \bar{X}[\bar{m}+1] = 1/2^n$ for the same reason.

The remaining cases follow by symmetry. This completes the proof.

B.4 Proof of the CM-Collision Bound (Lemma 3)

Proof: At the top level, we consider two cases: $N \neq \bar{N}$ and $N = \bar{N}$. The second of these will be analyzed by breaking into four subcases.

CASE 1: $N \neq \bar{N}$. In this case there are two ways for *bad* to be set to `true`: it can happen at line 31 or line 44 in the game that defines the CMcoll_n collision probability (Figure 4). Let us first calculate the probability that *bad* is set to `true` at line 31, which is

$$\Pr[\textit{bad} \text{ is set at line 31}] = \Pr[N \oplus L \in \{0^n,\ \bar{X}[1], \ldots, \bar{X}[\bar{m}+1]\}]$$

One point in the domain of π has been omitted from set $B = \{0^n,\ \bar{X}[1], \ldots, \bar{X}[\bar{m}], \bar{X}[\bar{m}+1]\}$: $\bar{X}[0] = \bar{N} \oplus L$, which we know is different from $N \oplus L$ since $N \neq \bar{N}$. The probability above is taken over L and $\bar{R}$, where each $\bar{X}[i]$ implicitly depends on both. We claim that for each of the $\bar{m}+2$ values in B, the probability that $N \oplus L$ is equal to this particular value is exactly $1/2^n$. This is verified by:

- $\Pr[N \oplus L = 0^n] = 1/2^n$ because of the random L.
- For any $j \in [1..\bar{m}-1]$, $\Pr[N \oplus L = \bar{X}[j]] = \Pr[N \oplus L = \bar{M}[j] \oplus \gamma_j \cdot L \oplus \bar{R}] = 1/2^n$ because of the random $\bar{R}$.
- Similarly, $\Pr[N \oplus L = \bar{M}[\bar{m}] \oplus (\gamma_{\bar{m}} \oplus \textit{huge}) \cdot L \oplus \bar{R}] = 1/2^n$ because of the random $\bar{R}$.
- Similarly, $\Pr[N \oplus L = \bar{M}[\bar{m}+1]] = \Pr[N \oplus L = \mathrm{Checksum}' \oplus \gamma_{\bar{m}} \cdot L \oplus \bar{R}] = 1/2^n$ because of the random $\bar{R}$.

We conclude that

$$\Pr[\textit{bad} \text{ is set at line 31}] \quad \leq \quad \frac{\bar{m}+2}{2^n} \tag{5}$$

We next show that

$$\Pr[X[c] \in \mathrm{Domain}(\pi) \text{ at line 40}] \quad \leq \quad \frac{c+\bar{m}+3}{2^n} \tag{6}$$

For this, let us define S to be

$$S = \{0^n,\ \bar{X}[0], \bar{X}[1], \ldots, \bar{X}[\bar{m}+1],\ X[0], X[1], \ldots, X[c-1]\}$$

This is the domain of π at the time that line 40 is executed. The set has $c+\bar{m}+3$ points and we shall use the sum bound to see that the probability that $X[m]$ is one of these is at most $(c+\bar{m}+3)/2^n$. Namely,

- $\Pr[X[c] = 0^n] = \Pr[\mathrm{len}(C[c]) \oplus (\gamma_m \oplus \textit{huge}) \cdot L \oplus R = 0^n] = 1/2^n$ as the right-hand side of the equality sign does not depend on R.
- $\Pr[X[c] = \bar{X}[0]] = \Pr[\mathrm{len}(C[c]) \oplus (\gamma_c \oplus \textit{huge}) \cdot L \oplus R = \bar{N} \oplus L] = 1/2^n$ for the same reason.
- For $j \in [1..\bar{m}-1]$, $\Pr[\mathrm{len}(C[c]) \oplus (\gamma_c \oplus \textit{huge}) \cdot L \oplus R = \bar{M}[j] \oplus \gamma_j \cdot L \oplus \bar{R}] = 1/2^n$ for the same reason.

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- $\Pr[X[c] = \bar{X}[\bar{m}]] = \Pr[\text{len}(C[c]) \oplus (\gamma_c \oplus \mathit{huge}) \cdot L \oplus R = \bar{M}[\bar{m}] \oplus (\gamma_{\bar{m}} \oplus \mathit{huge}) \cdot L \oplus \bar{R}] = 1/2^n$ for the same reason.
- $\Pr[X[c] = \bar{X}[\bar{m}+1]] = \Pr[\text{len}(C[c]) \oplus (\gamma_c \oplus \mathit{huge}) \cdot L \oplus R = \text{Checksum}' \oplus \gamma_{\bar{m}} \cdot L \oplus \bar{R}] = 1/2^n$ for the same reason.
- $\Pr[X[c] = X[0]] = \Pr[\text{len}(C[c]) \oplus (\gamma_c \oplus \mathit{huge}) \cdot L \oplus R = N \oplus L] = 1/2^n$ for the same reason.
- For $i \in [1..c-1]$, $X[i]$ is determined in one of two possible ways: either it is a value already placed into the Domain(π) (the **then** clause at line 37 was executed) or else it is a randomly selected value in $\{0,1\}^n$ (the **else** clause was executed). In the former case, the sum bound has already accounted for the probability of a collision with $X[i]$. In the latter case, the chance of the random value colliding with $X[c] = \text{len}(C[c]) \oplus (\gamma_c \oplus \mathit{huge}) \cdot L \oplus R$ is $1/2^n$.

Equation (6) has now been established.

Next we observe that

$$\Pr[X[c+1] \in \text{Domain}(\pi) \text{ at line } 44 \mid X[c] \notin \text{Domain}(\pi) \text{ at line } 40\,] \;\leq\; \frac{c+\bar{m}+4}{2^n} \tag{7}$$

The reason is that, when the conditioning event happens, $Y[c]$ is selected as a random point in $\{0,1\}^n$ at line 40, which results in Checksum being a random value independent of the points in the domain of π, which results in $X[c+1]$ being a random value independent of the points in the domain of π. Since the domain of π has at most $1 + (\bar{m}+2) + (c+1) = c + \bar{m} + 4$ points at this time, Equation (7) follows. Now, summing Equations (5), (6) and (7) gives us that

$$\Pr[\mathit{bad} \text{ gets set} \mid \text{Case 1}\,] \;\leq\; \frac{3\bar{m}+2c+9}{2^n} \tag{8}$$

Case 2A: $N = \bar{N}$ and $c \neq \bar{m}$. The next case we consider is when $N = \bar{N}$ and $c \neq \bar{m}$. Redefine S to be

$$S = \{0^n,\ \bar{X}[0], \ldots, \bar{X}[\bar{m}+1],\ X[1], \ldots, X[c-1]\}$$

This is Domain(π) at the time line 40 is executed. We show that

$$\Pr[X[c] \in S \mid \text{Case 2a}\,] \;\leq\; \frac{c+\bar{m}+2}{2^n} \tag{9}$$

To show this, one has as before to go through the $c + \bar{m} + 2$ points of S:

- $\Pr[X[c] = 0^n] = \Pr[\text{len}(C[c]) \oplus (\gamma_c \oplus \mathit{huge}) \cdot L \oplus R = 0^n] = 1/2^n$.
- $\Pr[X[c] = N \oplus L] = \Pr[\text{len}(C[c]) \oplus (\gamma_c \oplus \mathit{huge}) \cdot L \oplus R = N \oplus L] = 1/2^n$.
- For $j \in [1..\bar{m}-1]$, $\Pr[X[c] = \bar{X}[j]] = \Pr[\text{len}(C[c]) \oplus (\gamma_c \oplus \mathit{huge}) \cdot L \oplus R = \bar{M}[j] \oplus \gamma_j \cdot L \oplus R] = \Pr[\text{len}(C[c]) \oplus \bar{M}[j] = (\gamma_j \oplus \gamma_c \oplus \mathit{huge}) \cdot L] = 1/2^n$ since $\gamma_j \oplus \gamma_c \neq \mathit{huge}$. The reason that $\gamma_j \oplus \gamma_c \neq \mathit{huge}$ is that $\gamma_j < 2j \leq 2\bar{m} \leq 2 \cdot 2^{n-2} = 2^{n-1}$, so γ_j begins with a 0-bit; and $\gamma_c < 2c \leq 2\bar{m} \leq 2 \cdot 2^{n-2} = 2^{n-1}$, so γ_c begins with a 0-bit; so the xor of γ_j and γ_c begins with a 0-bit, while *huge* begins with a 1-bit, so they are certainly unequal.
- $\Pr[X[c] = \bar{X}[\bar{m}]] = \Pr[\text{len}(C[c]) \oplus (\gamma_c \oplus \mathit{huge}) \cdot L \oplus R = \text{len}(\bar{M}[\bar{m}]) \oplus (\gamma_{\bar{m}} \oplus \mathit{huge}) \cdot L \oplus R] = \Pr[\text{len}(C[c]) \oplus \text{len}(\bar{M}[\bar{m}]) = (\gamma_c \oplus \gamma_{\bar{m}}) \cdot L] = 1/2^n$ since $\gamma_c \neq \gamma_{\bar{m}}$ (since $c \neq \bar{m}$).
- $\Pr[X[c] = \bar{X}[\bar{m}+1]] = \Pr[\text{len}(C[c]) \oplus (\gamma_c \oplus \mathit{huge}) \cdot L \oplus R = \text{Checksum}' \oplus \gamma_{\bar{m}} \cdot L \oplus R] = \Pr[\text{len}(C[c]) \oplus \text{Checksum}' = (\gamma_c \oplus \mathit{huge} \oplus \gamma_{\bar{m}}) \cdot L] = 1/2^n$ as before.
- For $i \in [1..c-1]$, either $X[i]$ was selected as a value already in Domain(π), in which case the sum bound has already accounted for the probability of a collision with $X[c]$, or else $X[i]$ was selected as a new random value, in which case it has a $1/2^n$ chance of colliding with $X[c]$.

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We have established (9). Next, as before, if $X[c] \notin S$ then $Y[c]$ is chosen at random, making Checksum random, and making $X[c+1]$ random. Thus

$$\Pr[X[c+1] \in \text{Domain}(\pi) \text{ at line 44 } \mid X[c] \notin \text{Domain}(\pi) \text{ at line 40 }] \quad \leq \quad \frac{c+\bar{m}+3}{2^n} \tag{10}$$

since the size of the domain of π at line 44 is at most $c+\bar{m}+3$. Adding Equations (9) and (10) we have that

$$\Pr[\textit{bad} \text{ gets set } \mid \text{ Case 2A}] \quad \leq \quad \frac{2c+2\bar{m}+5}{2^n} \tag{11}$$

Case 2B: $N = \bar{N}$ and $c = \bar{m}$ and $\exists\, a,\ a < c$, s.t. $C[a] \neq \bar{C}[a]$. In this case, let $a \geq 1$ be the smallest index such that $C[a] \neq \bar{C}[a]$. We claim that $Y[a]$ is almost certainly not in the range of π when this point is examined at line 37, when $i = a$. In fact, we claim something stronger: that $Y[a]$ is almost certainly different from every point in

$$S = \{L,\ \bar{Y}[0], \ldots, \bar{Y}[c+1],\ Y[1], \ldots, Y[a-1],\ Y[a+1], \ldots, Y[c-1]\}$$

In particular,

$$\Pr[Y[a] \in S] \quad \leq \quad \frac{c+\bar{m}}{2^n} \tag{12}$$

This is verified by going through each point in S, exactly as before. This time, for each point in S except $\bar{Y}[a]$, the probability that this point coincides with $Y[a]$ is exactly $1/2^n$. The probability that $\bar{Y}[a] = Y[a]$ is 0, since $C[a] \neq \bar{C}[a]$.

Now we modify the game which defines CMcoll_n so that $X[a]$ is always selected at random from $\{0,1\}^n$. If we bound the probability that *bad* gets set in this new game and then add to it the bound of Equation (12), the result bounds the probability that *bad* gets set in Case 2B. From now on in this case analysis, assume this new game.

Next we claim that $X[c]$ is almost certainly different from $X[a]$:

$$\Pr[X[c] = X[a]] \quad = \quad \frac{1}{2^n} \tag{13}$$

This is clear because, in the modified game we have described, $X[a]$ is now chosen at random, independent of $X[c] = \text{len}(C[c]) \oplus (\textit{huge} \oplus \gamma_c) \cdot L \oplus R$.

We may now modify the game once again so that $Y[c]$ is selected at random even in the case that $X[c] = X[a]$. Bounding the probability of *bad* being set in the new game, and adding in the bound of (13), serves to bound the probability of *bad* being set in the prior game.

Now we can look at the probability that $X[c+1] \in \text{Domain}(\pi)$ when this is checked in the modified game. At this point the domain of π contains the $c+\bar{m}+3$ points

$$\text{Domain}^* \quad = \quad \{0^n,\ \bar{X}[0], \ldots, \bar{X}[\bar{m}+1],\ X[1], \ldots, X[a], \ldots, X[c]\}$$

We want to know the probability that $\text{Checksum} \oplus \gamma_c \cdot L \oplus R$ is in this set. But Checksum now contains the point $Y[c]$, which, in the modified game, has just been selected at random and independent of the points above. So

$$\Pr[X[c+1] \in \text{Domain}(\pi) \text{ in the modified game}] \quad \leq \quad \frac{c+\bar{m}+3}{2^n} \tag{14}$$

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Summing Equations (12), (13), and (14), we conclude that

$$\Pr[\mathit{bad} \text{ gets set } \mid \text{ Case 2B }] \leq \frac{2c + 2\bar{m} + 4}{2^n} \tag{15}$$

Case 2C: $N = \bar{N}$ and $c = \bar{m}$ and $C[i] = \bar{C}[i]$ for all $1 \leq i < c$ and $|C[c]| = |\bar{C}[c]|$. In this case, necessarily $C[c] \neq \bar{C}[c]$. Note that Checksum has a known value, which is different from Checksum$'$, being exactly Checksum$' \oplus \bar{C}[c]\, 0^* \oplus C[c]\, 0^*$. The values $M[1], \ldots, M[c-1]$ are likewise known, being identical to $\bar{M}[1], \ldots, \bar{M}[c-1]$, respectively. We are interested in

$$\Pr[X[c+1] \in \{0^n, X[0], \ldots, X[c], \bar{X}[c+1]\}$$

One goes through each of the points, as before, and sees that the probability that $X[c+1] = \text{Checksum} \oplus \gamma_c \cdot L \oplus R$ is any one of them is $1/2^n$, except for the last point, for which the probability that they coincide is 0. Thus

$$\Pr[\mathit{bad} \text{ gets set } \mid \text{ Case 2C }] \leq \frac{c+2}{2^n} \tag{16}$$

Case 2D: $N = \bar{N}$ and $c = \bar{m}$ and $C[i] = \bar{C}[i]$ for all $1 \leq i < c$ and $|C[c]| \neq |\bar{C}[c]|$. For this case, we first claim that $X[c]$ is almost certainly not in the domain of π when this is inspected at line 40 of Figure 4. The method is as before. The point $X[c]$ is certain to be different from $\bar{X}[c]$, and its chance of coinciding with any of the $c+2$ points in $\{0^n, X[0], X[1], \ldots, X[c-1], \bar{X}[c+1]\}$ is easily verified to be $1/2^n$. Thus

$$\Pr[X[c] \in \text{Domain}(\pi) \text{ at line 40 }] \leq \frac{c+2}{2^n} \tag{17}$$

Proceeding as before,

$$\Pr[X[c+1] \in \text{Domain}(\pi) \text{ at line 44 } \mid X[c] \notin \text{Domain}(\pi) \text{ at line 38 }] \leq \frac{c+3}{2^n} \tag{18}$$

since $c+3$ bounds the size of the domain when line 44 is executed, and the conditioning event ensures a random value for $X[c+1]$ which is independent of these points. Summing the bounds of Equation (17) and (18) gives

$$\Pr[X[c+1] \in \text{Domain}(\pi) \text{ at line 44 }] \leq \frac{2c+5}{2^n} \tag{19}$$

Conclusion. Taking the maximum from Equations (8), (11), (15), (16), and (19) we have

$$\Pr[\mathit{bad} \text{ gets set }] \leq \frac{3\bar{m} + 2c + 9}{2^n}$$

which is the lemma. ▮

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B.5 Proof of the Privacy Bound (Lemma 4)

The proof is straightforward compared to authenticity, so we quickly go though it. We begin by following the proof of the Structure Lemma (Appendix B.2). Games A to D are defined as before, except that

- The second half of each game is omitted, since there is no forgery attempt in this context.
- Return the truncated final-ciphertext-blocks, instead of the full final-ciphertext blocks, as the games specify.

Focus on the (modified) game C, where we have now returned to the adversary A a random string of $|M_r| + \tau$ bits whenever a query M_r is asked. Furthermore, the behavior of game C coincides with the behavior of the original game A unless the flag *bad* is set to `true`, at which point the two games diverge. Thus we can bound $\mathbf{Adv}^{\mathrm{priv}}_{\mathrm{OCB}[\mathrm{Perm}(n),\tau]}(A)$ by bounding the probability that the flag *bad* is set to `true` in (the modified) game C, which is at most the probability that it gets set in Game D. From the same reasoning as in the structure lemma, this is at most

$$\frac{(\sigma+2q+1)^2}{2^{n+1}} + \max_{\substack{m_1,\ldots,m_q \\ \sum m_i=\sigma \\ m_i\geq 1}} \left\{ \sum_{r\in[1..q]} \mathrm{Mcoll}_n(m_r) + \sum_{1\leq r<s\leq q} \mathrm{MMcoll}_n(m_r, m_s) \right\}$$

which is precisely the bound given by the the lemma. ∎

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